1 a. Find any critical numbers of the function \( f(x) = \sqrt[3]{4x - 6(x + 5)^4} \).
b. On what intervals is the function \( f \) increasing?

2. a. On what intervals is the function \( R(x) = \frac{2x}{5x^2 + 20} \) increasing?
b. Find the horizontal asymptote of \( R \).

3. a. Find the points of inflection of the function \( G(x) = (2x - 1)e^{2x} \).
b. On what intervals is the function \( G \) concave up?

4. Find all the critical points of function \( f \).
   \[ f(x) = x^{2/3}(2x - 5) \]

5. Find the absolute minimum and maximum values of function \( T \) on the given interval. \( T(\theta) = 2\cos \theta + \sin^2 \theta \) on the interval \( \left[ \frac{\pi}{2}, \frac{3\pi}{2} \right] \)

6. Find all values of \( x \) where the graph of \( g(x) = \ln(x^2 + 1) - 3 \) is increasing.

7. Find the absolute minimum and maximum values of function \( R \) on the given interval.
   \[ R(t) = \frac{t}{t^2 + 1} \] on the interval \([-4, 0]\)

8. Find all values of \( x \) for which the graph of \( f(x) = 27 + 8x^3 - x^5 \) is concave downward.

9. Find all local minima and maxima of \( g(x) = 3x^2e^{-4x} \).

10. Find any critical numbers of the function \( G \).
    \[ G(x) = (x - 4)^2(x - 7) \]

11. Find any critical numbers of the function \( f \). Also, record \( f' \) as a single rational function (i.e. find a common denominator.)
    \[ f(x) = x\sqrt{9 - 2x} \]
12. Find the points of inflection of the function \( T \) for \( 0 < x < \pi \). Upon which intervals is \( T(x) \) concave upwards?

\[
T(x) = \sin x + \cos x
\]

13. The path of a projectile thrown at an angle of 45° with level ground is

\[
y = x - \frac{32}{v_0^2} x^2
\]

where the initial velocity is \( v_0 \) feet per second. a) Find the x-coordinate of the point where the projectile strikes the ground. b) What is the instantaneous rate of change of height when the projectile is at its maximum height? c) What is the maximum height of the projectile?

Directions: Calculate the derivatives of the following functions.

14. \( f(x) = \left( \frac{x + 3}{x - 2} \right)^4 \)

15. \( g(x) = \frac{(2x + 1)^3}{(1 - 4x)^2} \)

16. \( h(x) = \arccos 3x^2 \)

17. \( T(x) = \sqrt{\frac{x - 8}{x + 4}} \)

18. \( f(y) = y^3 e^{5y} \sin y \)

19. \( g(x) = 4 \left( x^7 - 4x^2 \right)^3 \)

20. \( h(t) = \cos t^2 - t \)

21. \( T(w) = (w^2 - 9) \left( w^2 + w - 30 \right) \)

22. \( R(\theta) = \frac{\sqrt{7} \tan \theta}{1 - \theta} \)

23. \( W(t) = \sqrt{\sin \pi t} \)

24. An open box of maximum volume is to be made from a square piece of material, 16 inches on a side, by cutting equal squares from the corners and turning up the sides. Find the maximum volume.

25. A closed rectangular container with a square base is to have a volume of 2000 cubic inches. The material for the top and the bottom of the container will cost $2 per square inch. The material for the sides will cost $3 per square inch. Find the dimension of the container of least cost.
Directions: Find the indefinite integrals of the following functions.

26. \( \int \frac{1}{x^4} \, dx \)  
27. \( \int 8x^3 + 3x^2 + \pi \, dx \)

28. \( \int 1 + 2 \sin x \, dx \)  
29. \( \int x^2(x^3 + 5)^4 \, dx \)

30. \( \int x^3 \sqrt{x^4 + 6} \, dx \)  
31. \( \int \frac{x^3}{(1 + x^4)^2} \, dx \)

32. \( \int \sin \pi x \, dx \)  
33. \( \int x^2 e^{-x^3} \, dx \)

34. \( \int x^2 \left( x - \frac{8}{x} \right) \, dx \)  
35. \( \int \sin 2x \cos 2x \, dx \)

Directions: Evaluate the definite integrals.

36. \( \int_{-2}^{6} 3x \, dx \)  
37. \( \int_{1}^{4} x \sqrt{x} \, dx \)

38. \( \int_{0}^{4} \frac{1}{\sqrt{2x + 1}} \, dx \)  
39. \( \int_{-2}^{1} (2x + 1)^2 \, dx \)

40. \( \int_{-\pi/6}^{\pi/6} \sec^2 x \, dx \)  
41. \( \int_{-\pi/3}^{\pi/3} 4 \sec x \tan x \, dx \)

42. Use the Second Fundamental Theorem of Calculus to find \( F'(x) \).

\[
F(x) = \int_{-3}^{5x^2} 2t^3 - 7t \, dt
\]
Use L’Hôpital’s Rule to calculate the following derivatives.

43. \( \lim_{x \to 2} \frac{5 - \sqrt{7} + x}{x - 2} \)

44. \( \lim_{x \to 1} \frac{x^3 + x^2 - 2x}{x - 1} \)

45. \( \lim_{x \to 0} \frac{\ln 3x}{x} \)

46. \( \lim_{x \to \infty} \frac{3 - 2x}{x + 7} \)

47. \( \lim_{x \to \infty} \frac{x^3 - 3x^2 + 4}{x^4 - 4x^3 + 7x^2 - 12x + 12} \)

48. \( \lim_{t \to 0} \frac{2e^t - 2}{t} \)

49. \( \lim_{t \to 0} \frac{4^t - 6^t}{\sin 7t} \)

50. \( \lim_{x \to 0} \frac{e^x - x - 1}{x^2} \)

51. \( \lim_{w \to \infty} \frac{2\tan^{-1}(w)}{w} \)

52. \( \lim_{x \to \infty} x^3e^{-x^2} \)

53. \( \lim_{w \to \infty} \left(1 + \frac{1}{w}\right)^x \)

**Directions**: Determine whether the Mean Value Theorem can be applied to \( f \) on the closed interval \( [a, b] \). If the MVT can be applied, find all the values \( c \) in the open interval \( (a, b) \) such that \( f'(c) = \frac{f(b) - f(a)}{b - a} \). If the MVT can not be applied, explain why not.

54. \( f(x) = x^3, \ [0, 1] \)  

55. \( f(x) = x^4 - 8x, \ [0, 2] \)  

56. \( f(x) = \frac{x+1}{x}, \ [-1, 3] \)

57. \( f(x) = 3(10 + 4x)^{2/3} \) on \( [0, 2] \).  

58. \( T(x) = \tan x \) on \([\pi/4, \pi/4] \).

**Directions**: Find the value(s) of \( c \) guaranteed by the Mean Value Theorem for Integrals for the function over the given interval.

59. \( f(x) = \sqrt{x}, \ [4, 9] \)  

60. \( f(x) = \frac{9}{x^2}, \ [1, 3] \)
Directions: Calculate the derivatives of the following functions. Assume $A$ and $B$ are constants.

61. $f(y) = \frac{e^y}{y^3}$
62. $g(x) = \sqrt[5]{x^3} + 10\sqrt[5]{x^4}$

63. $h(t) = \frac{A}{t^8} + Be^{2t}$
64. $T(x) = 3\sec x + \tan x$

65. $R(y) = (y - 7)(e^y - \sqrt{y})$
66. $W(t) = \sin^2(5t)$

67. $y = \tan \pi x$
68. $y = 10x + 12\sqrt{x}$

69. $y = \ln \left[ x^5e^{2x} \right]$
70. $y = x5^x$

71. $y = \frac{3x}{1 + x^2}$
72. $e^x(x^3 + 4)$