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Cost and Production Duality With Time Utilization of Capital

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Abstract

This article fills in some notable gaps in the literature on the existence and empirical implementation of dual cost and production models embodying the time utilization of capital. A proof of the existence of such dual cost and production functions is provided; previous results of Betancourt (1986) and Klein (1984) are extended to the general N-input-factor, continuously variable time-utilization case; and the restrictive conditions under which a conventional neoclassical empirical cost model captures the characteristics of a capital-utilization technology are derived. The general specification of cost functions that capture utilization effects is indicated.

Key words: duality, capital utilization, time in production

JEL category: C81, D24, L23

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Interest in capital utilization issues both at the industry level (Beaulieu and Mattey 1998; Kim 1999; Paul 2003; Paul and Siegel 1999) and at the firm level (Maloney 2001; Powell and Schmenner 2002; Segerson and Squires 1995) has been growing, but despite gaps in the theoretical literature, little theoretical work has appeared recently. Winston (1982), for example, initially viewed cost and production duality applied to capital utilization and shiftwork with hostility, but shortly thereafter, limited duality results were extended to the capital utilization context by Betancourt (1986) and Klein (1984).[1]/ This literature struggled to specify production in such a way that the input intensity with which fixed capital is utilized at a point in time could be distinguished from variations in input usage due to variation in the period of time over which production occurs.[2]/

This article fills in some of the more notable gaps in the existing theoretical literature. This includes a formal general proof of the existence of dual cost and production functions embodying the time utilization of capital; extension of Betancourt's (1986) and Klein's (1984) results to the general N-input-factor, continuously variable time-utilization case; and derivation of the conditions under which a conventional neoclassical empirical cost model can capture the characteristics of a capital-utilization technology. When these conditions *do not* hold, then the utilization rate must be captured in the specification of output, inputs and input prices in order to avoid biased parameter estimates and inaccurate estimation of the characteristics of the underlying production technology. This last point is the missing key to empirical applications.

In addition to splitting the specification of output into time and rate of production components, input quantities and input prices also must reflect this distinction. Here, Georgescu-Roegen's (1970, 1971, 1972) flow-fund distinction is maintained in which labor and capital represent "funds" that are not immediately used up in the production process, while material and energy inputs are the "flows" that are either embodied in the product or discarded as waste.[3]/ Three types of input payment schemes are also maintained: prices invariant to time utilization that are applied to fixed quantities ("capital"); prices that are a function of utilization (shift or over-time premiums for "labor") applied to fixed or variable quantities; and fixed prices applied to quantities that accumulate with utilization ("materials and energy"). The flow-fund distinction affects the definition of capacity and its utilization, while the payment distinctions affect the incentives for high or low time utilization production plans. Oddly, Winston (1982) and Betancourt (1986) did not realize all of the resulting implications, because their two-input analytical models failed to embody the three input-payment methods, despite their forceful arguments toward that end.

Moreover, a formal general proof of the existence of dual cost and production functions embodying the time utilization of capital has not appeared to date. That proof, nearly trivial as it turns out, is offered here for a general N-input-factor, continuously variable time-utilization model of production. In short, the cost minimization problem with time utilization is identical in mathematical form to the traditional neoclassical cost minimization problem. The mathematical properties of neoclassical cost and production duality also carry through, although the interpretation of those properties changes to reflect the rate/time dichotomy explicit in the time utilization framework.

The resulting time-utilization cost function is used to examine the six propositions from the shiftwork literature demonstrated by Betancourt (1986) and Klein (1984).[4]/ As is typical of duality applications, the equivalent propositions follow from relatively simple manipulations of the derivatives of the cost function. Restrictions, other than limited continuity and convexity of the production set or function and differentiability of the cost function, are imposed in only one case and only in order to derive unambiguous effects. No specific functional forms are necessary.

The conditions under which a neoclassical empirical model can accurately capture the characteristics of production with time utilization are also considered. This is crucial for empirical work, as the presence of conditions under which time utilization can be safely ignored will greatly reduce the data requirements and econometric complexity for accurately estimating cost function parameters. On the other hand, the estimation of a neoclassical empirical model in the absence of these conditions will suffer from specification error and will yield biased and inconsistent parameter estimates as well as inaccurate estimates of elasticities of substitution and scale characteristics. Estimation of a properly specified flow-fund cost function, on the other hand, suffers from no such disabilities.

The application of duality theory to the analysis of capital utilization and shift-work solves several problems with the application of neoclassical economic analysis to these phenomena. Both Georgescu-Roegen (1970, 1972) and Winston (1982) point out that standard neoclassical cost and production theory, or its duality equivalent as formulated by Shephard (1970), ignores the duration component of economic processes as well as the administrative operation of the typical firm. Betancourt (1986) shows that ignoring the time utilization decision of the firm, as distinct from its choice of output quantity, leads to measurement error and simultaneity bias in the econometric estimation of the parameters of neoclassical cost functions.[5]/

It is shown here that only under restrictive conditions will the estimation of neoclassical flexible functional forms for cost or production functions accurately reflect the characteristics of the underlying production technology when time utilization varies across observations.[6]/ The cost and production duality derived here offers an empirically implementable means of addressing these issues.

The following section introduces the flow-fund production framework, reviews neoclassical duality theory, and proves the existence of cost and production duality with capital utilization. The six propositions are then examined in the general capital utilization duality model. Finally, the general conditions for neoclassical cost functions to accurately capture time utilization production technologies are considered.

1. Duality Theory with Capital Utilization

Georgescu-Roegen (1971) posits a "flow-fund" production model derived from process analysis of an idealized factory. The flow inputs are those that form a part of the finished product or exit the process as waste. These are typically the material and energy inputs to the process, the prices of which do not vary with the timing of production. [7]/ The fund inputs are present during all times of the actual production of output, but do not physically form part of the output nor exit the process as waste. These inputs are commonly called capital and labor.[8]/ The presence of the fund inputs along with a flow of other inputs produces a flow of output.

If the process operates continuously for a fixed period, say a "day", then a quantity of output, q, is produced. If the process operates for only t proportion of the day, then the total output produced is $t \cdot q$, where q is a function of the inputs:

(1) $$Q = t \cdot q = t \cdot g(k_1, k_2, \ldots, k_m, h_1, \ldots, h_r, x_1, \ldots, x_v) \qquad 0 \le t \le 1, \;\; m + r + v = N.$$

where Q is the total quantity of output produced in a "day"; t is the utilization rate or proportion of the day the process is in operation; q is the quantity of output produce by continuous operation of the process for the whole day, g is the production-rate function relating the quantities of the fund inputs (capital k_i, and labor h_j) present and the quantities of flow inputs (x_l) used in continuous production for the entire day to the output rate, q. Note that h_j represents the workers of a certain type present during production, not the number of employees. If two shifts are used then there are $2h_j$ employees.

The unique character of the flow-fund production model is in the specification of the input prices. The prices of the m capital inputs are fixed for any "day".[9]/ The prices of the v flow inputs are fixed per unit, regardless of when they are used.[10]/ The price of labor, however, is considered to vary with the utilization rate, either linearly as a fixed hourly wage or non-linearly to represent shift or overtime premiums, as in the shiftwork literature.[11]/

In the simple three-input case, the process operates for t proportion of the day with k capital and h labor present for that amount of time, and uses tx quantity of flow inputs. If the price of labor is a fixed wage rate, where the resulting price for the whole ($t = 1$) day is p_h, then labor is paid tp_h per unit. Thus, the cost of producing $Q = tq$ output in t ($0 < t \leq 1$) proportion of the day is $c = p_k k + (tp_h)h + p_x(tx)$. The price of labor may vary with t, $p_h = p_h(t)$, as long as the total price for any period t is known

Now consider cost and production duality in the traditional neoclassical framework as treated by Diewert (1978). Suppose we have an N factor production function F: $u = F(y_1, y_2,......,y_N) = F(y)$ where u is the amount of output produced during a given period of time and $y = (y_1,...., y_N) \geq 0_N$ is a non-negative vector of input quantities used during the period. Suppose also that the producer faces fixed positive prices for inputs $(p_1, p_2,...,p_N) \equiv p$ and does not possess market power in the input markets.

Define the producer's cost function C as the solution to the problem of minimizing the cost of producing at least output level u, given the input prices p, or

$C(u, p) = \min_y \{p'y : F(y) \geq u \}$.

The following Assumption is sufficient to imply the existence of solutions to the cost minimization problem and this is stated as a Lemma.

Assumption 1: F is continuous from above; i.e., for every $u \in$ range F,

$L(u) \equiv \{y : y \geq 0_N, F(y) \geq u \}$ is a closed set.

Lemma 1: If F satisfies Assumption 1 with $p >> 0_N$, then for every $u \in$ range F,

$\min_y \{p'y : F(y) \geq u \}$ exists.

Furthermore, the cost function has certain well known properties[12]/ which imply Shepherd's Lemma, as follows:

Lemma 2: If the cost function satisfies the properties in footnote 13 and is differentiable with respect to input prices at the point (u*, p*), then

$y(u^*, p^*) = \nabla_p \ C(u^*, p^*)$

where $y(u^*, p^*) \equiv [y_1(u^*, p^*), y_2(u^*, p^*),....,y_N(u^*, p^*)]'$ is the vector of cost minimizing input quantities needed to produce u* units of output given input prices p*, where the underlying production function F is defined above, $u^* \in$ range F and $p^* >> 0_N$. This lemma establishes the differentiability property of the cost function by which the input factor demand functions can be derived. One need only start with a cost function satisfying the appropriate properties to derive results consistent with the production function F.

Now consider a flow-fund production function G :

$Q = G(k_1, k_2,..., k_m, h_1,..., h_r, X_1,..,X_v) \equiv tg(k_1, k_2,..., k_m, h_1,..., h_r, x_1,..,x_v) = tq$,

where Q is the amount of output produced in t proportion of a given period of time, $0 \leq t \leq 1$, and $z = (k_1,...,k_m, h_1,..., h_r, X_1,..., X_v) \geq 0_N$, $m+r+v = N$, is a non-negative vector of fund (k_i, h_j) and flow ($X_l = tx_l$) inputs.[13] Given positive prices for the inputs, $P = (P_1,.., P_m, P_{m+1},...,P_{m+r}, P_{m+r+1},...,P_{m+r+v})$, where $P_j = p_j(t)$ for $(m + 1) \leq j \leq (m + r)$, then the flow-fund equivalent of Lemma 1 is the following.

Lemma 1': If G satisfies Assumption 1 with $P >> 0_N$, then for every $Q \in$ range G,

$\min_z \{P'z : G(z) \geq Q\}$ exists.

PROOF: G is a production function with the same properties as the production function F for Lemma 1. Given a vector of positive input prices, the proposition follows directly by application of Lemma 1 for G satisfying Assumption 1.

Thus, the flow-fund cost function $C(Q, P) = \min_z \{P'z : G(z) \geq Q\}$ not only exists, but has the usual properties. If it is also differentiable, then Shepherd's Lemma (Lemma 2) holds.[14]/ This is all that is needed to confront the generalized propositions of capital utilization. [15]/

Nevertheless, this does not justify estimation of a conventional neoclassical cost function in the presence of utilization effects. The flow-fund cost function embodies utilization effects through the specification of input prices as functions of the utilization rate. As will become clear below, failure to correctly specify and estimate input prices will produce biased parameter estimates and inaccurate characterization of the underlying production technology.

2. The Six Propositions

Each of these propositions concerns the behavior of the cost function as the utilization rate, or degree of shift work, changes. To evaluate them, it is helpful to examine the elasticity of the cost function with respect to the utilization rate, t. Using differentiability of the cost function and Shepherd's Lemma,

(2) $$E_{Ct} = (t/C)C_t \Big|_{dQ=0}$$

$$= [C_Q(\partial Q/\partial t) + \sum_{i=m+1}^{r+v} C_i(\partial P_i/\partial t) + \sum_{j=m+r+1}^{m+r+v} P_j(\partial C_j/\partial t)](t/C)$$

where C_j represents the partial derivative of cost with respect to variable j, or, in the case of inputs, the price of input j. Note that the partial derivative of cost with respect to t is evaluated at dQ = 0. Thus, C_Q represents the change in C as q changes to keep Q constant as t changes, Q = tq. Or, dQ = qdt + tdq = 0 => dq/dt = -q/t < 0. It is more revealing to write C_q as the change in cost caused by changing the rate of production to keep Q constant as t changes. Moreover, $C_q \geq 0$ by the non-decreasing in output property of cost functions. Then the first term inside the bracket in (2) becomes

$$C_q(-q/t)(t/C) = -C_q(q/C) = -E_{Cq}$$

where E_{Cq} is of the elasticity of cost with respect to the rate of production, q.

For the second term inside the bracket, $\sum_{i=m+1}^{m+r} C_i(\partial P_i/\partial t)(t/C) = \sum_{i=m+r}^{N} S_i e_{it}$, where $S_i = (P_iC_i)/C$ is the cost share of input i and $e_{it} = P_{it}(t/P_i)$ is the elasticity of input price P_i with respect to the utilization rate t.[16]/ Similarly, $P_j(\partial C_j/\partial t)(t/C) = P_j x_j(t/C) = S_j$. Thus, (2) collapses to

(3) $$E_{Ct} = -E_{Cq} + \sum_{i=m+1}^{m+r} S_i e_{it} + \sum_{j=m+r+1}^{m+r+v} S_j \quad .$$

From (3), we can see immediately the N-factor equivalents of Betancourt's propositions 1, 2, 3 and 5. These are restated below using the notation B1,...,B6 to distinguish Betancourt's original propositions from the more general forms derived here and designated K1, etc.

Proposition K1(B1): The cost of higher utilization increases as the elasticity of any input price with respect to utilization (e_{it}) increases.

This folllows from inspection of (3).

Proposition K2(B2): The larger the degree of economies of scale, the higher the cost of high utilization.

In (3), $-E_{Cq}$ is the scale effect, where $E_{Cq} = 1$ for constant returns to scale and $E_{Cq} < 1$ for economies of scale. The smaller E_{Cq} , the greater the economies of scale and the smaller negative is the scale effect on E_{Ct}, and the larger E_{Ct} becomes as t increases. Thus, economies of scale decrease the incentive to choose high utilization.

Proposition B3: If the production function is homothetic, higher utilization systems use more capital intensive processes than do lower utilization systems.

Proposition B5: For the generalized Leontief cost function, the higher the capital intensity of the technology the lower the costs of the high utilization system.

The price of capital inputs does not increase with utilization by definition. The "non-capital" inputs are those whose price (labor) or usage (materials) increases with utilization. The greater the "capital intensity" of the process, the larger the share of the capital inputs and the smaller the shares of the non-capital inputs in cost. Only the shares of the "labor" inputs appear in (3). The smaller the shares of those inputs, the smaller is E_{Ct}, and the lower the cost of high utilization (t). Conversely, high utilization processes must have higher "capital intensity" (capital's share of cost) than lower utilization processes, other things equal. Thus propositions B3 and B5 are consolidated for the general N-factor case as:

Proposition K3: For a differentiable cost function, the higher the share of the capital inputs in cost, the lower the cost of increased utilization.

Now consider Betancourt's propositions 4 and 6. Both of these propositions involve the reaction of utilization to a change in the price of labor and its relationship to the elasticities of substitution and economies of scale. Specifically, higher elasticities of substitution of capital for labor are associated with higher utilization.

Proposition B4: Under constant returns to scale, a decrease in the relative price of labor increases (decreases) the incentive to utilize when the elasticity of substitution is less (greater) than unity.

Proposition B6: For the generalized Leontief cost function with only two inputs, capital and labor, the higher the *ex ante* elasticity of substitution between capital and labor, the lower the costs of the high utilization system.

In addition to imposing constant returns to scale for B4 and the Leontief functional form for B6, Betancourt also used the Allen-Uzawa form of the elasticity of substitution in deriving these propositions. The Allen-Uzawa form has since been largely discredited in favor of the Morishima elasticity of substitution (Blackorby and Russell).

To update and extend Betancourt's analysis, consider the change in the relative cost shares of two inputs as utilization changes in a simple two-input-factor model in which factor 1 is "labor" and factor 2 is "capital".

$$\begin{aligned}&\partial(S_1 / S_2) / \partial t = \partial(P_1 C_1 / P_2 C_2) / \partial t \\ &= (C_1 / P_2 C_2)[P_{1t} + (P_1 C_{11} P_{1t} / C_1) + (P_1 C_{1q} q_t / C_1) - (P_1 C_{21} P_{1t} / C_2) - (P_1 C_{2q} q_t / C_2)]\end{aligned}$$

Then, using the definitions of the own-price, cross-price, and output elasticities of input demands,

$$\varepsilon_{ii} = C_{ii}(P_i / C_i) \qquad \varepsilon_{ij} = C_{ij}(P_i / C_j) \qquad \varepsilon_{iq} = C_{iq}(q / C_i)$$

and that $q_t = -q/t$, we find:

(4) $\partial(S_1 / S_2) / \partial t = (C_1 / P_2C_2)[P_{1t}(1 + \varepsilon_{11} - \varepsilon_{12}) - (P_1 / t)(\varepsilon_{2q} - \varepsilon_{1q})]$

$$= (C_1 / P_2C_2)[P_{1t}(1 - M_{12}) - (P_1 / t)(\varepsilon_{2q} - \varepsilon_{1q})]$$

where $M_{12} = \varepsilon_{12} - \varepsilon_{11} = (P_1C_{12} / C_2) - (P_1C_{11} / C_1)$ is the Morishima elasticity of substitution[17]/, which appears due to the upward slope of labor's price in utilization space. The sign of (4) depends on whether the elasticity of substitution is greater or less than one, and on the relative magnitudes of the output elasticities of demand for the two inputs. In fact, if the production technology is homogeneous of any degree or homothetic, then $\varepsilon_{1q} = \varepsilon_{2q}$ and the sign of (4) depends only on the elasticity of substitution. If the elasticity of substitution of labor for capital (that is, when labor's price changes) is greater than one, then labor's share of cost declines relative to capital as utilization increases, the expected result in high utilization processes.

For the two-factor homothetic case examined by Betancourt, and the homothetic case of N factors with only one factor ("labor") price a function of utilization, we have the following.

Proposition K4: The higher the *ex ante* (Morishima) elasticity of substitution between labor and non-labor inputs, the greater the incentive for high utilization.

This Proposition may also hold under less restrictive conditions, but no general result can be derived. To see this, consider any labor (i) and non-labor (j) input pair in the N-factor case, for which we can derive a similar result, although one lacking in intuitive appeal:

$$\partial(S_i / S_j) / \partial t = (C_i / P_jC_j)[P_{it}(1 - M_{ij}) + (P_i / t)(\varepsilon_{jq} - \varepsilon_{iq}) + P_i(\sum_{h\neq i}(C_{ih}P_{ht} / C_i) - \sum_{h\neq j}(C_{jh}P_{ht} / C_j)]$$

If the sign of this expression is negative, then labor's relative share declines as utilization increases and E_{Ct} also declines in (4), increasing the incentive for higher utilization. Alternatively, if labor's price increases for fixed t, this causes a decrease in labor's share of cost relative to capital if $1-M_{ij} > 0$, or $0 < M_{ij} < 1$. Consequently, E_{Ct} also declines and the incentives for higher utilization increase. Moreover, if only one of the N input prices is a function of t, then each $P_{ht} = 0$, the summation terms vanish, and equation (5) results.[18]/

Then for the general N-factor case, we have:

Proposition K5: An increase (decrease) in the price of labor relative to the price of a non-labor input, t fixed, decreases (increases) the incentive for high utilization, if the (Morishima) elasticity of substitution is less than one.

3. When Does Utilization Matter?

When the neoclassical model is adequate, then capital utilization can be ignored in estimating the properties of the production technology; if the neoclassical model is lacking, capital utilization must be accounted for in order to properly characterize the production technology. As this is an empirical econometric question, rather than one of pure theory, consider the three-input neoclassical Cobb-Douglas cost function, C_N, and the corresponding three-input flow-fund, or capital utilization, Cobb-Douglas cost function, C_F, specified below in empirical log-linear forms.

(5) $\ln C_N = a + b(\ln p_K) + c(\ln p_H) + d(\ln p_X) + e(\ln Q) + u$

(6) $\ln C_F = \alpha + \beta(\ln P_K) + \gamma(\ln P_H) + \delta(\ln P_X) + \varepsilon(\ln t) + \eta(\ln q) + \mu$

where a,b,c,d,e,α, β, γ, δ, ε, and η are parameters to be estimated, u and μ are error terms, and labor's price in the flow-fund formulation, $P_H = p_h(t)$, is a function of utilization. First, note the obvious, that if t = T, a fixed constant, then equation (6) collapses to equation (5), as (Tq) substitutes for both Q and q, and all input prices are exogenous.[19]/

If one were to use cross sectional data to estimate equation (5) when the true equation is (6), it is immediately apparent that $\ln p_H$ is correlated with the error term through the missing variable, t, leading to biased and inconsistent parameter estimates. The econometric solution to this problem is the well-known instrumental variables technique, which can be applied given observation of the utilization rate to allow estimation of labor's price as a function of t. The neoclassical model does not observe t and so must fail in this case.

The second problem in estimating (6) with (5) is specification error from substituting lnQ for lnt and lnq, which generally results in biased parameter estimates. It is worth asking, however, whether Q can capture the relevant variation in t and q under some conditions. When this is true, then

$$e(\ln Q) = e(\ln t + \ln q) = e(\ln t) + e(\ln q) = \varepsilon(\ln t) + \eta(\ln q)$$

which requires $e = \varepsilon = \eta$, or that the effect on cost of variations in t is the same as the effect on cost of variations in q. In more general terms, this can be written as

$$(7) \qquad \frac{\partial \ln C}{\partial \ln t} = \frac{\partial \ln C}{\partial \ln q} = \frac{\partial \ln C}{\partial \ln Q}$$

or that the cost function is homogeneous of the same degree in t and q. Further, if a cost function is homogenous of degree α, then its dual production function is homogeneous of degree (1/α). [20]/ For a three-input flow-fund production technology of the form in equation (1), equation (7) requires

$$(8) \qquad \lambda^{1/\alpha} G(k, h, X) = (\lambda t)g(k,h,x) = tg(k\lambda, h\lambda, x\lambda)\,.$$

Since the flow-fund production technology in equation (1) is linearly homogeneous in t, it must also be linearly homogeneous in the inputs that determine q, in order for the neoclassical formulation to capture the characteristics of this technology ($1/\alpha = \alpha = 1$).

Furthermore, if equation (7) holds, then equation (3) collapses to

$$E_{Ct} = -E_{Cq} \leq 0\,.$$

If this holds as a strict inequality, a corner solution results in which all producers choose t=1 and variations in output occur through variations in q alone. If the equality holds, then producers are indifferent concerning t, as the same costs result from all levels of t for any given output quantity Q.[21]/ In either case, the neoclassical cost and production framework is adequate, as the distinction between t and q has no effect on cost.

These results are stated as a proposition.

Proposition 6: The neoclassical cost and production model accurately captures the characteristics of flow-fund (capital utilization) production technology when

a) the utilization rate, t, does not vary such that t = T, a constant.

or

b) all input prices are independent of utilization, t, *and*

the dual flow-fund cost and production functions are homogeneous of the same degree in utilization, t, as in rate of production, q.

As the requirements in Proposition 6 pertain to observed production, it is difficult to assess their likelihood as well as the effects of their presence or absence without empirical evidence. Nevertheless, we observe instances in which they will not hold: when the time utilization period of production varies and when input prices – such as overtime or shift-work premiums for labor or peak-load pricing of energy inputs – vary by time of day or utilization rate. In these instances, a production model that explicitly recognizes capital utilization is required to accurately characterize the underlying technology. The flow-fund models explored here indicate the appropriate empirical specification in such cases.

4. Conclusion

A dual cost function exists for the flow-fund production technology incorporating capital utilization and has all the properties associated with cost functions dual to neoclassical production technologies. The proof is trivial, but has not previously appeared. The existence of cost and production duality for models with capital utilization greatly facilitates further analysis of the determinants of the degree of observed capital utilization. These models restrict the definitions of outputs, inputs and input prices that lead to accurate estimation of production characteristics when the utilization rate varies.

Further, five of the six propositions of the shift-work literature identified by Betancourt (1986) are shown to hold in slightly modified forms for the unrestricted N-input-factor flow-fund technology with capital utilization, and the sixth holds under more general conditions than Betancourt identified. Unlike previous attempts, these results are derived without restricting the form of the production technology or the number of inputs. The results nevertheless retain their intuitive appeal. A high degree of capital utilization is associated with a low responsiveness of input prices to changes in utilization; a low degree of economies of scale; a high cost share for capital; and a high elasticity of substitution between labor and non-labor inputs.

The final result may be the most important and the most useful: identification of the conditions under which neoclassical empirical production models can characterize flow-fund technologies with capital utilization. The first of these is the obvious situation in which there is no observed variation in capital utilization. If variation in utilization exists, then a neoclassical model still may be adequate if input prices are independent of utilization and the underlying flow-fund technology is homogeneous of the same degree with respect to utilization, t, as it is to rate of production, q. If these conditions are not satisfied, then utilization must be taken into account explicitly as shown here in order to accurately characterize the production technology.

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[1] / Segerson and Squires (1995) derive similar results for a revenue-maximizing firm.

[2] / This was motivated in part by Marris's (1964) finding that despite the apparent cost incentives for multiple shift production - fixed costs could be spread over more units - many establishments in non-continuous process industries operated for substantially less than a full 24-hour day. In fact, Clark (1923) had anticipated the importance of the ensuing literature at the establishment level. Whether this micro-level significance carries through to the macro-level remains an issue of some controversy (Veracierto 2002).

[3] / Smith (1961) specified a similar stock/flow dichotomy, but classified labor as a flow. Beard and Lozada (1999) offer a cogent summary of Georgescu's approach.

[4] / Betancourt states these as: 1) the higher the shift (wage) differential, the higher the costs of the multiple-shift system and the lower the incentive to utilize; 2) the larger the degree of economies of scale, the higher the costs of the double-shift system; 3) higher utilization systems use more capital intensive processes than do lower utilization systems; 4) a decrease in the relative price of labor increases (decreases) the incentive to utilize, if the elasticity of substitution is less (greater) than unity under constant returns to scale; 5) the higher the capital intensity of the technology and 6) the higher the *ex ante* elasticity of substitution between capital and labor, the lower the costs of the high utilization system. Betancourt (1986) used a two-input-factor, discrete (i.e., one versus two shifts) production specification, assuming homotheticity for propositions 2) through 4) and the generalized Leontief functional form for 5) and 6). Klein (1984) derived the n-factor equivalents of propositions 1, 2, 4, and 5 assuming only homotheticity of the production rate function.

[5] / Betancourt offers several econometric approaches to estimating capital utilization cost functions, either by stratifying the sample observations by degree of capital utilization or by estimating instruments for the variables

affected by the capital utilization decision. Estimation of short run cost or profit functions, in which the fixed capital input is measured by its "capacity," offers an intuitively pleasing way to capture utilization variations through implicit output:capacity ratios. Nevertheless, if input prices vary with utilization (shift or overtime premiums, peak-load pricing, etc.), then instrumental variables, multi-stage least-squares, or other technique must be employed to obtain accurate characterizations of the underlying technology.

[6] / As we shall see, two firms using the same technology , one running one shift and the other two, will appear to have different labor:capital ratios, even though the numbers of workers present per machine at any time may be the same. If labor's price (wage rate) varies by shift or time of day, then failure to explicitly capture its slope will underestimate the firm's marginal cost and distort relative efficiency measures.

[7] / Peak-load pricing of energy or other inputs are ignored for simplicity at this stage, but can be easily added to the model if required.

[8] / Chemical catalysts might also qualify fund inputs. Georgescu-Roegen (1971) also identifies the "process-fund" consisting of one potential piece of output at each stage of the process which is necessary before output may exit the production process.

[9] / Some capital inputs could be leased or rented only for the period the process operates each day. Certain maintenance or depreciation costs related to capital may increases with utilization. These possibilities can be modeled in the same way that the price of labor is specified below. The fixed price, on the other hand, is typical of land and structures.

[10] / Obviously, peak-load pricing and inventory costs and methods might violate this assumption. These special cases can be modeled in this framework similar to the way labor's price is specified here, but are ignored for simplicity.

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11 One might also consider these groups classified as to the method of payment: k for fixed price fund inputs, x for fixed price-per-unit flows, and h for inputs whose price varies with utilization.

12 / Specifically, the cost function has the following seven properties: 1) C is non-negative; 2) C is (positively) linearly homogeneous in input prices for any level of output; 3) If any combination of input prices increases, then the minimum cost of producing any feasible output level u will not decrease; 4) for every u $\in$ range F, C(u, p) is a concave function of p; 5) for u $\in$ range F, C(u, p) is continuous in p, $p >> 0_N$; 6) C(u, p) is nondecreasing in u for fixed p; 7) for every $p >> 0_N$, C(u, p) is continuous from below in u.

13 / This implicitly assumes that all tq combinations are feasible. This may be unrealistic. For example, some or all production rates may not be physically achievable for very short periods of time. As long as such time periods are well below the range of observed variation, this is not likely a problem in practice. If only a small number of discrete tq combinations are feasible, however, then the continuity of the production technology is threatened and dual cost functions may not exist or not be differentiable.

14 / Differentiability and continuity of C and G in q and t imply that all combinations of q and t are feasible. This condition may not hold in some commonly observed production processes. One cannot necessarily produce one 240th the quantity of steel in one minute that one produces in four hours due to indivisibilities in the duration required to accomplish certain metallurgical transformations.

15 / By similar arguments, dual variable profit and variable cost functions also exist, from which variable net-put equations can be derived (Klein 1980).

16/ This requires multiplication by one in the form (P_i / P_i) and rearranging terms.

17 / Compare this to the Allen-Uzawa elasticity of substitution, $\sigma_{ij} = (C_{ij} C)/C_i C_j$). The Morishima formulation is not symmetric for any two inputs i and j as is the Allen-Uzawa, but it does provide an exact measure

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of curvature and ease of substitution, as well as complete comparative static information on relative factor shares (Blackorby and Russell).

[18] / The summation terms represent the net effect of the proportional changes in the demands for inputs i and j in response to changes in the prices of other “labor” inputs as t changes.

[19] / It is also obvious that the flow-fund production function G collapses to the neoclassical production function F under this condition.

[20] / See Chambers (1988), p. 76.

[21] / The non-decreasing in output property of cost functions requires the inequality.

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