DEPARTMENT OF ECONOMICS AND FINANCE WORKING PAPERS SERIES • SEPTEMBER 2008

Inferences for Selected Location Quotients with Applications to Health Outcomes

Gemechis D. Djira[1], **Frank Schaarschmidt**[2], **Bichaka Fayissa**[3]

[1] *Department of Mathematics and Statistics, South Dakota State University, SD*

[2] *Institute of Biostatistics, Leibniz University of Hannover, Germany*

[3] *Department of Economics and Finance, Middle Tennessee State University, TN*

Abstract

Location quotient (LQ) is an index frequently used in geography and economics to measure the relative concentration of activities. This quotient is calculated in a variety of ways depending on which group to use as a reference. Here, we focus on simultaneous inference for the ratios of the individual proportions to the overall proportion based on binomial data. Apparently, this is a multiple comparison problem and multiplicity adjusted location quotients have not been addressed up to now. In fact, there is a negative correlation between the comparisons. The quotients can be simultaneously tested against unity and simultaneous confidence intervals can be constructed for the LQs based on existing probability inequalities and by directly using the asymptotic joint distribution of the associated z-statistics. The proposed inferences are appropriate for analysis based on sample surveys. A real data set is used to demonstrate the application of multiplicity adjusted LQs. A simulation study is also carried out to assess the performance of the proposed methods in terms of achieving a nominal coverage probability. It is observed that the coverage of the simple Bonferroni adjusted Fieller intervals for LQs is just as good as the coverage of the method which directly takes the correlations into account.

Key Words: Location quotients, Fieller's theorem, Multiple comparison

JEL Category: C12, R11

[1] Gemechis D. Djira, Assistant Professor Department of Mathematics and Statistics, South Dakota State University, Harding Hall 123, Brookings, SD 57007, Phone: 605-688-6878, Fax: 605-688-6814; Email: gemechis.djira@sdstate.edu

1 Introduction

One of the tools for comparing area features of the local economy relative to the regional or national economic activity is the location quotient (LQ). Owning to its simplicity, LQ has extensively been used by regional economists and economic geographers for making the comparison of an area's share of some activity with the share of some base aggregate such as employment, manufacturing, retail services, mortgage loans, health care services, etc. (Isard, 1960; Thrall, et al., 1995). The LQ measure has primarily been applied in the economic base studies (Cortese and Leftwich, 1975; Isserman, 1977; Wright, 1994; Bogart and Ferry, 1999), relying on input-output data (Flegg, et al., 1995; McCann and Dehurst, 1998) for quantifying and comparing the local concentration of economic activities relative to the national economy.

Typically, a location quotient compares the proportion of employment (or income) in a particular industry (employment category) within the local economy (e_i/e) to the proportion of employment (or income) in that same industry within a larger reference economy (E_i/E), i.e. $\text{LQ}_i = (e_i/e)/(E_i/E)$. Based on the assumption that a typical urban economy is a microcosm of the national economy, Leigh (1970) calculates LQ for explaining whether the activities of a given urban economy is similar (LQ $=$ 1), specialized (LQ $>$ 1), or non-specialized (LQ $<$ 1) to the national economy. According to Silocks (1994), Moineddin, et al. (2003), and Beyene and Moineddin (2005), this index can also be used to quantify and compare health outcomes across spatial domains. Simply stated, a location quotient can be defined as a ratio of location parameters (e.g., ratios of means or proportions). In many applications, location quotient is known as ratios of individual proportions to proportion of a reference group. For example, the reference group can be the overall proportion of the proportions being compared or proportion of a different standard group. Depending on the choice of the reference group, LQ is computed differently. Our primary focus is on inferences for location quotients computed as ratios to the overall proportion in a sample survey with binomial counts. An important issue in this type of comparisons is that the LQs are negatively correlated, and one needs to account for these correlations. Another commonly observed drawback of LQ (in sample surveys) is its widespread application as only a point estimate without an accompanying confidence interval. Beyene and Moineddin (2005) discuss methods of constructing marginal confidence intervals for the LQs. Our main objective is to develop methods of constructing multiplicity adjusted Fieller (1954) confidence intervals which more accurately reflect the variability as well as the correlations in LQs.

The methods will be used to construct multiplicity adjusted infant mortality rate (IMR) location quotients for the 10 standard metropolitan statistical areas (SMSAs) in the State of Tennessee as a case study. Infant mortality rate is one of three variables for constructing a composite index known as Human Development Indicator (HDI). Economic development policy makers are interested in knowing the variations in IMR across metropolitan areas in order to prudently allocate health care services such as pre-natal care and the need to make health insurance affordable for working families in different localities.

The rest of the paper is organized as follows. In Section 2, we present methods of making simultaneous inferences for LQs. In Section 3, the methods will be demonstrated on a real dataset. Section 4 deals with a simulation study to assess coverage probabilities of the proposed simultaneous confidence intervals for LQs. We give concluding remarks in the last section.

2 Inferences for location quotients

Let Y_{ij}, $j = 1, ..., n_i$ be a random sample of binary responses from the i^{th} group having success probability of π_i, $i = 1, ..., k$. The groups can be industries, metropolitan areas, counties, etc. It is assumed that $Y_{i.} = \sum_{j=1}^{n_i} Y_{ij} \sim$ Binomial(π_i, n_i). The parameters of interest are the ratios of the success probabilities of the individual groups to the success probability of all the groups combined. Let $N = \sum_{i=1}^{k} n_i$ denote the total number of observations, and let $\overline{\pi} = \sum_{i=1}^{k} \frac{n_i}{N} \pi_i$ be the weighted (or pooled) population proportion. Here, location quotient is defined as

$$\lambda_i = \frac{\pi_i}{\overline{\pi}}, \quad i = 1, \ldots, k. \tag{1}$$

If the location quotient of a group is 1, it means that the group has a share similar to the reference group (i.e., all groups combined). A location quotient of less (greater) than 1 implies that the group has less (greater) share than the reference group .

In subsequent sections, we develop methods of simultaneously making inferences about the population location quotients (λ_i).

2.1 Simultaneous confidence intervals

Simultaneous confidence intervals can be derived for λ_i, $i = 1, \ldots, k$ by utilizing Fieller (1954) theorem in conjunction with the joint distribution of some relevant test statistics. Let $P_i = Y_{i.}/n_i$ and $\overline{P} = \sum_{i=1}^{k} \frac{n_i}{N} P_i$ denote unbiased estimators of π_i and $\overline{\pi}$, respectively. Let $L_i = P_i - \lambda_i \overline{P}$, $i = 1, \ldots, k$. For large sample sizes, $L_i \sim N(0, \sigma^2_{L_i})$, where

$$\sigma^2_{L_i} = \mathrm{var}(L_i) = \left(1 - 2\lambda_i \frac{n_i}{N}\right) V_i + \lambda_i^2 \sum_{h=1}^{k} \left(\frac{n_h}{N}\right)^2 V_h, \tag{2}$$

and $V_i = \mathrm{var}(P_i) = \pi_i(1 - \pi_i)/n_i$. Let $Z_i = L_i/\widehat{\sigma}_{L_i}$, where $\widehat{\sigma}^2_{L_i}$ is the same as (2) with V_i replaced by $\widehat{V}_i = P_i(1 - P_i)/n_i$. The random vector $(Z_1, \ldots, Z_k)'$ has an approximate k-variate normal distribution with zero vector of means and correlation matrix $\mathbf{R}_1[a_{ij}]$, where $a_{ij} = \mathrm{cov}(L_i, L_j)/(\sigma_{L_i}\sigma_{L_i})$, and

$$\mathrm{cov}(L_i, L_j) = \lambda_i \lambda_j \sum_{h=1}^{k} \left(\frac{n_h}{N}\right)^2 V_h - \lambda_i \frac{n_j}{N} V_j - \lambda_j \frac{n_i}{N} V_i, \tag{3}$$

$1 \leq i \neq j \leq k$. Now, marginal or simultaneous two-sided confidence interval estimates of λ_i can be obtained by solving the equation

$$\frac{L_i}{\widehat{\sigma}_{L_i}} = q \tag{4}$$

for λ_i, $i = 1, \ldots, k$. The solutions are the same as that of Fieller (1954) intervals, except that we get marginal or simultaneous confidence intervals depending on how the quantile q is specified (for example, see Dilba et al., 2006; Djira and Hothorn, 2008). If q is the $(1 - \frac{\alpha}{2})^{\mathrm{th}}$ quantile of a univariate standard normal distribution, Equation (4) yields marginal Fieller confidence intervals for λ_i. And if q is multiplicity adjusted critical point, one gets simultaneous confidence intervals. The simplest multiplicity adjusted critical point is Bonferroni adjustment in which case q is the $(1-\frac{\alpha}{2k})^{\text{th}}$ quantile of the standard normal distribution. An adjustment which directly takes the correlations into account is to take q as an equi-coordinate percentage point of the distribution of $(Z_1,\ldots,Z_k)'$. Since $\mathbf{R}_1$ depends both on the unknown location quotient parameters (λ_i) and the proportions (π_i), we plug in maximum likelihood estimates of λ_i and π_i in $\mathbf{R}_1$. Thus, an approximate equi-coordinate percentage point of size α, say $C_{\alpha,\widehat{\mathbf{R}}_1}$, is determined such that

$$\text{prob}\{-C_{\alpha,\widehat{\mathbf{R}}_1}\leq Z_i\leq C_{\alpha,\widehat{\mathbf{R}}_1},\quad i=1,\ldots,k\}\approx 1-\alpha.$$

We call this method the "plug-in" approach. Yet, another method based on Sidak (1967) inequality consists of replacing $\mathbf{R}_1$ by the identity matrix $\mathbf{I}_k$ for computation of critical points. In general, the last method is slightly less conservative than Bonferroni adjustment.

2.2 Hypothesis testing

Consider the elementary hypotheses

$$\mathrm{H}_{0i}:\ \lambda_i=1\quad\text{versus}\quad \mathrm{H}_{1i}:\ \lambda_i\neq 1,\quad i=1,\ldots,k.$$

It is of interest to test the union-intersection hypothesis given by

$$\mathrm{H}_0:\ \bigcap_{i=1}^{k}\mathrm{H}_{0i}\quad\text{versus}\quad \mathrm{H}_1:\ \bigcup_{i=1}^{k}\mathrm{H}_{1i}. \tag{5}$$

The alternative hypothesis H_1 states that at least one of the location quotients is different from 1 (i.e., less than 1 or greater than 1). Under H_0, $L_i=P_i-\lambda_i\overline{P}$, is distributed approximately as normal with mean zero and variance

$$\sigma^2_{L_i}=\overline{\pi}(1-\overline{\pi})\left[\frac{1}{n_i}-\frac{1}{N}\right],\quad i=1,\ldots,k.$$

Under H_0, the joint distribution of $Z_i=L_i/\widehat{\sigma}_{L_i}$, $i=1,\ldots,k$, is approximately k-variate normal with zero vector of means and correlation matrix $\mathbf{R}_0[b_{ij}]$, where

$$b_{ij}=\frac{-1}{\sqrt{\left(\frac{N}{n_i}-1\right)\left(\frac{N}{n_j}-1\right)}}, \tag{6}$$

$1 \leq i \neq j \leq k$. If the sample sizes are equal (i.e., $n_i = n$, $i = 1, \ldots, k$), b_{ij} reduces to $-1/(k-1)$. For the test problem in (5), multiplicity adjusted critical point of size α, say $C_{\alpha,\mathbf{R}_0}$, is determined as a two-tailed equi-coordinate percentage point of the joint distribution of the Z_is. The null hypothesis H_0 will be rejected if $|Z_i| > C_{\alpha,\mathbf{R}_0}$, for at least one i, $i = 1, \ldots, k$. The associated two-tailed adjusted p-values can be calculated as

$$\widetilde{p}_i = 1 - \text{prob}\{-|z_i| \leq Z_1 \leq |z_i|, \ldots, -|z_i| \leq Z_k \leq |z_i|\}, \quad i = 1, \ldots, k,$$

where z_is are the observed values of the test statistics (see, for example, Westfall et al., 1999, for the computation of adjusted p-values for other multiple testing problems).

For both multiple testing and simultaneous confidence interval estimations, the related quantiles and probabilities of multivariate normal distribution can be calculated, for example, in R software (R Development, 2007) using the extension package called *mvtnorm* (see, Hothorn et al., 2001).

3 Example

In this section, simultaneous inferences for locations quotients is performed for a data on infant mortality rates in 10 standard metropolitan statistical areas (SMSAs) in the State of Tennessee, USA in 2001. Table 1 summarizes the data, the calculated Z statistics, and the adjusted and unadjusted p-values for testing the location quotients against unity (See Figure 1 for the plot of the adjusted p-values on the ten metropolitan areas). The application of multiplicity adjusted methods results in broader confidence intervals than applying the unadjusted methods. Figure 2 shows that there is practically no difference between the limits based on Bonferroni adjustment and the Plug-in method.

Based on the unadjusted methods, one would conclude LQs to be significantly smaller than one for Cleveland, Knoxville, and Nashville and significantly larger than one for Memphis. However, taking the multiplicity adjustment into account, we find significant effects only for the latter two. With simultaneous confidence level of 95%, we can state that the infant mortality rate of Memphis is between 110% and 153% of the average infant mortality in

Table 1: Observations, estimates, test statistics and p-values for simultaneously testing $\mathrm{H}_{0i} : \lambda_i = 1,\ i = 1, \ldots, 10$ (Tennessee metropolitan areas, 2001)

	Observations					p-value	p-value
Metropolitan area	n_i	$y_{i.}$	P_i	$\widehat{\lambda}_i$	Z_i	(Unadj.)	(Adj.)
Chattanooga	4539	43	0.009	1.04	0.25	0.801	1.000
Clarksville	2353	20	0.008	0.93	−0.33	0.742	1.000
Cleveland	1369	7	0.005	0.56	−1.58	0.114	0.680
Jackson	1482	22	0.015	1.63	2.34	0.019	0.172
Johnson-City	1978	20	0.010	1.11	0.47	0.641	1.000
Kingsport-Bristol	2320	24	0.010	1.13	0.63	0.531	0.999
Knoxville	7563	50	0.007	0.72	−2.48	0.013	0.122
Memphis	15568	187	0.012	1.32	4.44	0.000	0.000
Morristown	1592	14	0.009	0.96	−0.14	0.885	1.000
Nashville	17519	127	0.007	0.79	−3.16	0.002	0.016

Tennessee's Metropolitan areas. In Knoxville, it ranges between 45% and 100%, while it is 62% and 97% in Nashville.

4 Simulation study

The proposed confidence intervals are based on a large sample approximation. It is known that large sample approximations of binomials may perform poorly when applied in situations with extreme proportions (i.e., π_is close to 0 or 1) and small to moderate sample sizes. We performed a simulation study to explore the small sample performance of our methods and show the limitations of their application for small sample sizes.

In simultaneous estimation of several parameters, the coverage probability of the confidence intervals is of interest, i.e., the probability that the set of confidence limits covers all true parameters. We estimated the simultaneous coverage probability for the unadjusted confidence intervals, the Bonferroni, Sidak, and Plug-in method based on 10000 random samples from four bi-

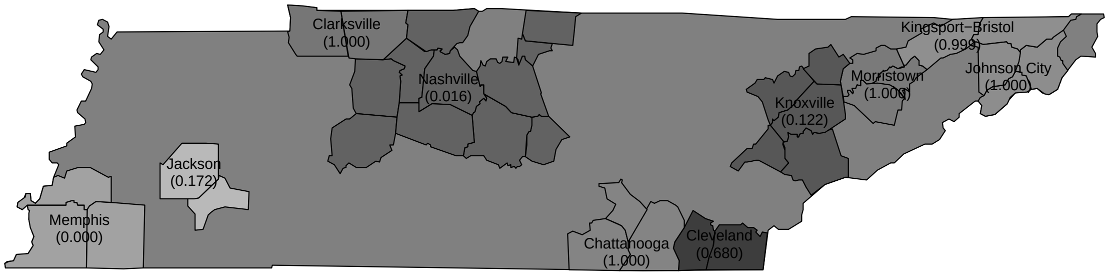

Figure 1: Map of the 10 metropolitan statistical areas of Tennessee. Grey shades indicate whether the infant mortality LQs are less than 1 (dark grey) or higher than 1 (light grey). The values in parentheses are multiplicity adjusted p-values for testing $\mathrm{H}_{0i} : \lambda_i = 1$, $i = 1, \ldots, 10$.

nomial distributions (i.e., $k = 4$) for a number of parameter settings. We also considered the following three sample size configurations. (a) Balanced: $n_i = 50$, $i = 1, 2, 3, 4$, (b) balanced: $n_i = 100$, $i = 1, 2, 3, 4$, and (c) unbalanced: $n_1 = 50, n_2 = 100$, $n_3 = 500, n_4 = 1000$. In Table 2, empirical simultaneous coverage probabilities are shown for different choices of π_i. The number of simulation steps for which finite-width Fieller confidence intervals are obtained is given in the last column of Table 2. Here, the simultaneous coverage probability is defined as the probability that all true parameters are covered by the set of confidence intervals, given the confidence intervals are bounded.

First, using methods without multiplicity adjustment lead to serious inflation of the type-I error. This is illustrated by the sixth column in Table 2. Comparing the four groups with exactly equal proportions to their overall proportion will lead to erroneous identification of a difference in proportion in about one third of the cases when it is intended only in 10% of the cases. This probability of erroneous identifications of deviations from the common proportion will increase as the number of groups increases. Second, the proposed methods to construct simultaneous confidence intervals perform equally well when sample sizes are large and proportions are intermediate. Remarkably, the computationally simple Bonferroni adjustment performs as

Table 2: Empirical coverage probabilities for a nominal level of $1-\alpha=0.90$

Parameter settings					Coverage probability				Simulation
n_i	π_1	π_2	π_3	π_4	Unadju.	Bon.	Sidak	Plug-in	used
a	0.1	0.1	0.1	0.1	0.659	0.870	0.856	0.842	9784
	0.1	0.1	0.1	0.2	0.651	0.855	0.853	0.843	9840
	0.1	0.1	0.1	0.8	0.633	0.834	0.828	0.809	9838
	0.1	0.1	0.2	0.2	0.653	0.862	0.860	0.855	9897
	0.1	0.1	0.5	0.5	0.644	0.861	0.855	0.844	9907
	0.1	0.1	0.8	0.8	0.654	0.855	0.851	0.838	9896
	0.1	0.2	0.5	0.8	0.655	0.869	0.863	0.855	9946
	0.2	0.2	0.2	0.2	0.661	0.879	0.879	0.866	9997
	0.5	0.5	0.5	0.5	0.672	0.898	0.894	0.890	10000
	0.5	0.5	0.8	0.8	0.666	0.890	0.888	0.876	10000
b	0.1	0.1	0.1	0.1	0.661	0.880	0.876	0.873	10000
	0.1	0.1	0.1	0.2	0.669	0.879	0.879	0.872	10000
	0.1	0.1	0.1	0.8	0.651	0.855	0.854	0.844	10000
	0.1	0.1	0.2	0.2	0.679	0.888	0.885	0.878	9999
	0.1	0.1	0.5	0.5	0.671	0.880	0.877	0.868	9999
	0.1	0.1	0.8	0.8	0.660	0.870	0.865	0.857	9999
	0.1	0.2	0.5	0.8	0.669	0.886	0.881	0.871	10000
	0.2	0.2	0.2	0.2	0.681	0.892	0.888	0.882	10000
	0.5	0.5	0.5	0.5	0.687	0.906	0.902	0.895	10000
	0.5	0.5	0.8	0.8	0.682	0.901	0.897	0.890	10000
c	0.1	0.1	0.1	0.1	0.672	0.873	0.869	0.856	9945
	0.1	0.1	0.1	0.2	0.662	0.870	0.865	0.847	9952
	0.1	0.1	0.1	0.8	0.656	0.871	0.864	0.828	9956
	0.1	0.1	0.2	0.2	0.667	0.870	0.866	0.844	9942
	0.1	0.1	0.5	0.5	0.677	0.881	0.875	0.840	9953
	0.1	0.1	0.8	0.8	0.656	0.876	0.870	0.831	9958
	0.1	0.2	0.5	0.8	0.680	0.897	0.890	0.855	9941
	0.2	0.2	0.2	0.2	0.681	0.890	0.888	0.876	9999
	0.5	0.5	0.5	0.5	0.696	0.908	0.905	0.893	10000
	0.5	0.5	0.8	0.8	0.681	0.896	0.893	0.883	10000

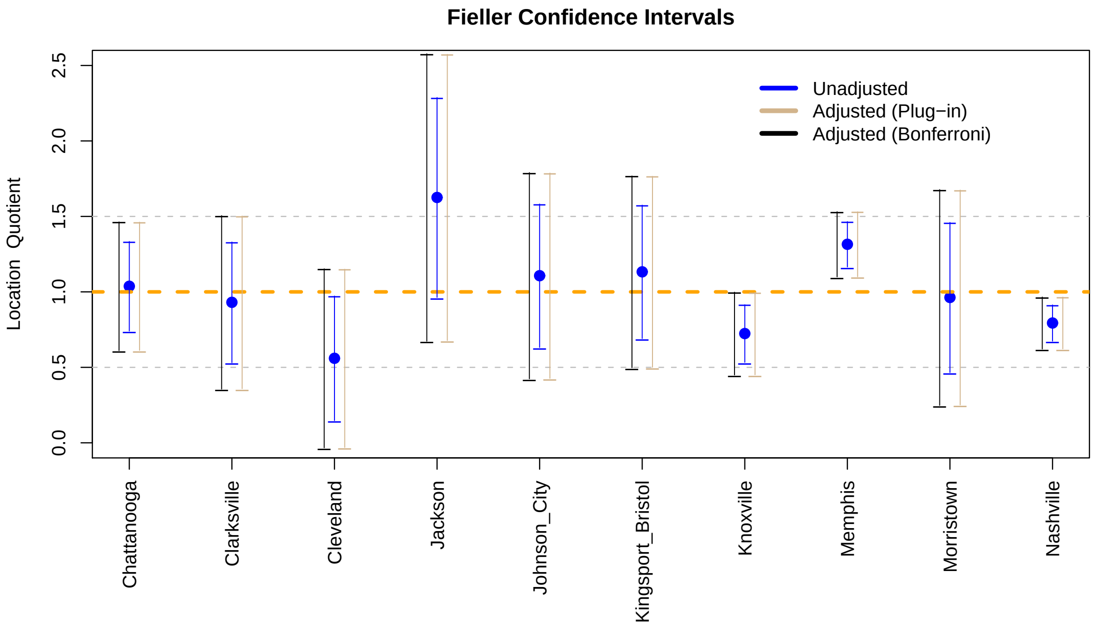

Figure 2: Adjusted and unadjusted 95% Fieller confidence intervals for location quotients.

good or better than the computationally more demanding methods. For extreme proportions (i.e. proportions close to 0 or 1) and small sample sizes, all methods should be used with caution since in extreme cases, they might show coverage probabilities as low as 81% when nominal confidence level is 90%. However, this result occurs for a situation where rather small proportions ($\pi_i = 0.1$) simultaneously occur with rather large proportions $\pi_i = 0.8$, a situation which is probably rare in practical applications.

5 Conclusion

In this paper, methods for simultaneous inference of multiple location quotients are discussed with focus on binomial samples. By their definition, the location quotients considered here lead to a problem of multiple comparisons. Confidence intervals can be constructed using Fieller's theorem and can be adjusted for multiplicity either by using simple probability inequality or methods based on multivariate normal quantiles. In a simulation study, the simultaneous coverage probability of different methods is investigated for situations with moderate sample sizes. First, this study illustrates that estimating confidence intervals without adjustment for multiple comparisons leads to inflated probabilities of type-I error. Second, the multiplicity adjusted methods show empirical simultaneous coverage probabilities close to the nominal level in settings with balanced as well as unbalanced sample sizes. Finally, all methods considered have simultaneous coverage probabilities slightly lower than the nominal level for the majority of the settings with the simple Bonferroni adjustment performing best. This liberal performance is more pronounced when proportions are extreme and hence normal approximation becomes inaccurate.

For the simpler problem of comparing only two binomial proportions, Gart and Nam (1988) consider Fieller-type intervals among other methods based on Taylor series expansion and iterative approaches. They show that Fieller-type intervals have acceptable small sample properties compared to simpler Taylor series approaches, but can be improved by iterative approaches. However, the iterative approaches are computationally more intensive and their adaptation to multiple location quotients appears to be not straightforward. Dann and Koch (2005) show for the two-sample comparison that Fieller-type confidence intervals have empirical coverage probabilities close to or slightly higher than the nominal level. For methods based on Taylor series expansion, they show a violation of the nominal level.

In conclusion, the proposed methods take into account both the variability and the correlations in location quotients defined as ratios to the overall proportion. Therefore, inferences based on multiplicity adjustments provide a more statistically sound results in judging the significance of locations quotients.

References

Beyene, J. and R. Moineddin (2005). "Methods for Confidence Interval Estimation of a Ratio Parameter with Application to Location Quotients." *BMC Medical Research Methodology* 5, 32.

Bogart, W. T. and W. C. Ferry (1999). "Employment Centres in Greater Cleveland: Evidence of Evolution in a Formerly Monocentric City." *Urban Studies* 36, 2099-2110.

Cortese, C.F. and J.E. Leftwich (1975). "A Technique for Measuring the Effect of Economic Base on Opportunity for Blacks." *Demography* 12, 325-329.

Dann, R.S. and G.G. Koch (2005). "Review and Evaluation of Methods for Computing Confidence Intervals for the Ratio of Two Proportions and Considerations for Non-inferiority Clinical Trials." *Journal of Biopharmaceutical Statistics* 15, 85-107.

Dilba, G., F. Bretz, and V. Guiard (2006). "Simultaneous Confidence Sets and Confidence Intervals for Multiple Ratios." *Journal of Statistical Planning and Inference* 136, 2640-2658.

Djira, G.D. and L.A. Hothorn (2008). "Detecting Relative Changes in Multiple Comparisons with an Overall Mean." *Journal of Quality Technology* (in press).

Fieller, E.C. (1954). "Some Problems in Interval Estimation." *Journal of the Royal Statistical Society, Series B* 16, 175-185.

Flegg, A. T., C. D. Webber, and M. V. Elliott (1995). "On the Appropriate Use of Location Quotients in Generating Regional Input-Output Tables." *Regional Studies* 29, 547-61.

Gart, J.J. and J. Nam (1988). "Approximate Interval Estimation of the Ratio of Binomial Parameters: A Review and Corrections for Skewness." *Biometrics* 44, 323-338.

Hothorn, T., F. Bretz, and A. Genz (2001). "On Multivariate t and Gauss Probabilities." *R News* 1(2), 27-29.

Isard, W. (1960). *Methods of Regional Analysis.* New York: John Wiley & Sons.

Isserman, A. M. (1977). "The Location Quotient Approach to Estimating Regional Economic Impacts." *Journal of the American Institute of Planners* 43, 33-41.

Leigh, R. (1970). "The Use of Location Quotients in Urban Base Studies." *Land Economics* 46, 202-205.

McCann, P., and J. H. Dewhurst (1988). "Regional Size, Industrial Location and Input-Output Expenditure Coefficients." *Regional Studies* 32, 435-44.

Moineddin, R., J. Beyene, and E. Boyle (2003). "On the Location Quotient Confidence Interval." *Geographical Analysis* 35, 249-256.

R Development Core Team (2007). *R: A Language and Environment for Statistical Computing. R Foundation for Statistical Computing.* Vienna, Austria. ISBN 3-900051-07-0, URL http://www.R-project.org.

Silcocks, P. (1994). "Estimating Confidence Limits on a Standardized Mortality Ratio When the Expected Number is Not Error Free." *Journal of Epidemiology & Community Health* 48, 313-317.

Thrall, G., J. Fandrich, and S.E. Thrall (1995). "Location Quotient: Descriptive Geography for the Community Reinvestment Act." *Geo Info Systems*, 5(6), 18-22.

Westfall, P.H., R.D. Tobias, D. Rom, R.D. Wolfinger, and Y. Hochberg (1999). *Multiple Comparisons and Multiple Tests Using the SAS System.* Cary, NC, SAS Institute Inc, pp. 65-81.

Wright, S. E. (1994). "The Spatial Distribution and Geographic Analysis of Endodontic Office Locations at the National Scale." *Journal of Endodontics* 20, 500-505.