Income Inequality, Monetary Policy, and the Business Cycle

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Abstract

The effects of changes in monetary policy are studied in a general equilibrium model where money facilitates transactions. Because there are two types of agents, workers and capitalists, different elasticities of money demand exist, implying that monetary policy influences the distribution of income. When earnings inequality is incorporated into monetary policy rule is the model able to replicate cyclical fluctuations of both real and nominal aggregates as well as the inequality measure. Additionally, monetary policy becomes more countercyclical when the fraction of transfers received by the workers increases.

Key words: Inflation, Income Distribution, Heterogenous Agents, Perturbation

JEL category: E32; E42; E50.

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1 Introduction

The dynamic effects of monetary policy are documented in a model where inflation has differential impacts on segments of the population. There are several motivations. First, there is evidence suggesting that the poor tend to only hold money while the rich diversify (Guiso, Haliassos, and Jappelli, 2002). Therefore, it is hoped that by quantifying the effects of monetary policy in a heterogenous agents model one can potentially show how policy affects the distribution of income and hence inequality. Second, Bernanke (2003) implies that accommodative monetary policy can help reduce the inequality that arises when the benefits of productivity increases are mainly attributed to firm owners. Thus, agent heterogeneity allows for the modeling of monetary policy as a function of inequality. Third, the research of Sims (1980, 1999) and Bernanke and Mihov (1998) find no support that the parameter weights for monetary policy rules have changed even though policy's covariance structure changed – policy became more countercyclical after 1979. Therefore, modeling a relationship between policy and inequality allows for the quantification of the effects of *intensive* monetary policy rule changes¹ that might arise from, for example, a distributional change in inequality. Indeed, there is substantial empirical evidence that the distribution of income between the rich and poor has changed (see Katz and Autor 1999 for a review).

In the first part of the paper, the empirical business cycle relationship between monetary policy and inequality is documented for the U.S. economy. The seigniorage tax rate, that is to represent monetary policy, and cyclical Gini coefficient are found to have a negative contemporaneous correlation. However, the lagged Gini coefficient significantly and positively affects current monetary policy in a reduced form regression. The estimated impulse response functions also imply that positive innovations in inequality are associated with increases in the next period's money stock. Interestingly, the converse is not true; innovations in monetary policy have a negative but small effect on inequality. The interpretation given is that monetary policy systematically responds over the business cycle to deviations in income inequality with a lag. Finally, though seigniorage became more countercyclical after 1979, the parameters of the regression are constant (stable) with respect to time.

In the second part of the paper, the quantitative results of the modeling are presented. A key assumption of the analysis is that there are two types of agents in the model economy: capitalists and workers. Additionally, as in Kydland (1991) and Gavin and Kydland (1999), money offers a time-saving role thereby altering the labor-leisure trade-off. As a result,

¹Intensive monetary policy changes are to be where the distribution of the policy variable changes but not the weights in the policy rule.

the capitalists will hold both capital and fiat money even though capital's rate of return dominates money. The workers, who cannot enter the capital markets, save only through fiat money balances implying that there exist two different elasticities for money demand.² A final feature of the model is that the seigniorage generated revenues are allowed to be transferred back to the agents. The primary purpose of this assumption is to capture the potential distributional effects of inflation that are not in the model. For example, "tax bracket creep" from unanticipated inflation has been found to redistribute income between the rich and poor (Heer and Süssmuth, 2003).

There are several theoretical results. First, when the monetary policy rule is exogenous to inequality then the relationship between income inequality and monetary policy does not resemble the observed pattern for the U.S. economy. Alternatively, when the monetary policy feedback rule is modeled as endogenous to earnings inequality, the relationship between the seigniorage tax rate and inequality can be replicated. That is, an increase in earnings inequality will increase the rate of money growth and, thus, increase transfers. As transfers increase income inequality falls giving a negative correlation between the Gini coefficient and seigniorage. Because there is persistence in earnings inequality, an increase in income inequality will imply increases in the next period's seigniorage tax rate – the lagged Gini coefficient positively affects current monetary policy.

Second, temporary and exogenous monetary expansions are found to decrease inequality through redistribution. However, the effects are temporary and small.³ Rather, welfare is mainly affected by the consumption smoothing effects of tying policy to inequality. The lifetime consumption of the worker is made smoother which increases their elasticity for labor effort. The cost from the resulting destabilized economy get passed through to the capital owners. The welfare effects of long-run inflation critically depends on the cause of the inflation. For example, when the fraction of seigniorage-generated transfers received by the workers is low, inflation is smaller but workers are worse off. Alternatively, exogenous and permanent increases in the seigniorage rate can increase the welfare of the poor; the mean of lifetime utility increases for the workers.

Finally, change in the composition of the types of households receiving the seigniorage generated transfers – an intensive monetary policy rule change – is found to alter the cyclical properties of nominal aggregates. Specifically, the rate of money creation becomes more

²Though the partition is extreme, so is the concentration of capital for industrialized countries. For example, over the 1983-98 time period only 40.5% of all U.S. households held risky assets (Guiso, Haliassos, and Jappelli 2002). Similarly, only 17.0% of German and 17.5% of Italian households hold risky assets.

³Romer and Romer (1999) find similar empirical results where temporary expansionary monetary policy slightly increases the welfare of the poor.

procyclical when the fraction of transfers received by the workers falls even though the parameters of the monetary rule are stable. Because the model is highly stylized with respect to the transfers, however, the question of why the effects of inflation on transfers changed in the direction of the working poor cannot be answered. The cause for this intensive policy change is thus a direction for future research.

The rest of this paper is organized as follows. Section 2 examines the empirical relationship between income inequality and monetary policy. Section 3 defines the model economy. Section 4 characterizes the equilibrium. Section 5 contains details of the computational algorithm and the model calibration. The results and conclusion are presented in Sections 6 and 7, respectively.

2 The Empirical Relationship Between Inequality and Monetary Policy

It has long been acknowledged that unexpected inflation (via monetary policy) acts to affect the distribution of income (Hubbard 2002, chapter 28). There are several channels to which income distribution can be influenced. First, inflation taxes the poor more heavily since they hold a larger fraction of their wealth in money. Second, unanticipated inflation transfers income from nominal lenders to nominal borrowers. Because governments frequently borrow to expenditures which include the funding of social programs, the poor are likely nominal borrowers (albeit indirectly). Third, Heer and Süssmuth (2003) find empirical evidence that tax bracket creep from inflation reduces income inequality. Finally, government transfers, that are included by the Bureau of the Census in their computation of the Gini, are funded in part by revenues generated by seigniorage. It is the current practice in the United States for the Federal Reserve to pay for its operating expenses and then rebate all other revenues to the Treasury; in this case the government and monetary authority are said to pool revenue. Additionally, Click (1998) reports that seigniorage finance accounts for about 2% of average U.S. government spending. Thus, pooling can be a significant and direct way for income to be redistributed when spending is directed towards lower incomes.

Because inflation can theoretical have either positive or negative effects on inequality, there are two main questions addressed in this section. First, how are income inequality and monetary policy related over the business? Second, do policy makers understand this link and, hence, respond to deviations in inequality? Towards this end, annual U.S. data are collected from four sources: the IMF's International Financial Statistics, the FRED database at the Federal Reserve Bank of St. Louis, an inequality data set from the U.S. Bureau of the Census, and the Bureau of Labor Statistics. The series for real GDP, CPI, and population are identified from the IFS data set. The log of real per capita GDP and inflation, denoted y_t and π_t respectively, are then computed in standard fashion. From the FRED, an annual per capita monetary base (denoted m) is constructed by averaging the monthly per capita monetary base. Then the growth rate, or seigniorage tax rate, is computed from the following definition: the growth in per capita monetary base: $\theta_t = (m_t - m_{t-1})/m_{t-1}$. The Gini coefficient from the U.S. Bureau of the Census is defined formally as $gini_t = 1 - 2 \int_0^1 L(y_t) dy_t$, where $L(y_t)$ is the Lorenz curve of the income distribution. The unemployment rate, u_t , is selected from the Bureau's employment status of the civilian noninstitutional population data series.

For computation of the correlation coefficients, the Hodrick-Prescott (HP) filter is employed where the smoothing parameter is set to 6.25 following the discussion in Ravn and Uhlig (1997). For the regression models, the first differences, denoted with a " Δ ", are used instead. Finally, the years for the sample must be set. Though the Bureau of the Census has updated the inequality series, unfortunately its definition of income changed significantly after 1992. Thus, comparisons of the Gini computed before and after 1993 are difficult. Therefore, the sample years are set at 1965-1992.

Table 1 reports that the seigniorage tax rate and Gini coefficient averaged 6.37% and 35.5%, respectively, over the sample period. Additionally, their business-cycle components are slightly procyclical. However, because there are indications that monetary policy was altered around 1979, the sample is divided into two sub-samples. Panels B and C of Table 1 show that, prior to 1979, the monetary policy variable is procyclical. Alternatively, the seigniorage rate is slightly countercyclical after 1979. The cyclical Gini shows no significant changes over the two samples. An equality of variance test indicates that only the seigniorage rate is significantly different after the break; the variances of the Gini and GDP do not appear to change.

The business-cycle components of the seigniorage tax rate and the Gini coefficient are negatively correlated at $corr(\theta_t, gini_t) = -20.95\%$; the first panel of Figure 1 illustrates the negative relationship. A negative correlation is consistent with Romer and Romer (1999), and the papers cited within, who also find a negative contemporaneous time series link between income inequality and monetary policy. However, the second panel of Figure 1 is the scatter plot for the tax rate and the lag of the Gini coefficient. There appears to be a positive link – in terms of lags – between the two variables; $corr(\theta_t, gini_{t-1}) = 36.26\%$.

Figure 1: Historical Comparison of Seigniorage Tax Rate and U.S. Gini Coefficient (Note: HP-filtered values).



	Trend	Std Dev%	Correlation	Correlation		
			with $y_t \%$	with $gini_{t-1}\%$		
Panel A: 1965-1992						
$ heta_t$	0.062	0.96	15.53	36.26		
$gini_t$	0.315	0.22	5.02	10.96		
y_t	10.035	2.01	1.0	42.47		
u_t	0.061	0.71	-70.29	-50.92		
π_t	0.054	1.47	-13.72	-11.42		
Panel B: 1965-1979						
$\overline{ heta_t}$	0.078	0.72	63.28	72.89		
$gini_t$	0.304	0.20	-4.92	-29.29		
y_t	9.956	2.21	1.0	45.98		
u_t	0.054	0.74	-74.76	-42.33		
π_t	0.057	1.54	-24.78	-13.34		
Panel	C: 1980-	- <u>1992</u>				
$ heta_t$	0.069	1.21	-26.18	14.53		
$gini_t$	0.328	0.25	10.40	41.85		
y_t	10.120	1.72	1.0	37.63		
u_t	0.069	0.67	-60.48	-60.08		
π_t	0.051	1.42	8.40	-7.88		
Panel D: Equality of Sub-Sample Variances (Folded-F)						
$\hat{\theta}_{i}$ $\hat{F}_{(10,14)} = 2.85^{**}$						

 Table 1: Sample Statistics of HP-Filtered Series

θ_t	$F_{(12,14)} = 2.85^{*}_{(0.065)}$
$gini_t$	$\hat{F}_{(12,14)} = \frac{1.60}{_{(0.400)}}$
y_t	$\hat{F}_{(14,12)} = \underset{(0.386)}{1.66}$
u_t	$\hat{F}_{(14,12)} = \frac{1.21}{_{(0.747)}}$
π_t	$\hat{F}_{(14,12)} = \underset{(0.781)}{1.18}$

Note: *p*-values in parentheses; **Significant at 10%.

In support, the first panel in Table 2 reports the estimations for a series of *reduced-form* regressions using the differenced data and the Generalized Method of Moments (GMM). The equations chosen for the model include all the variables except unemployment; instrumental variables will be inoperable since unemployment and GDP are highly collinear.⁴ The estimations confirm that the lag of the Gini coefficient significantly affects the seigniorage tax rate at a 10% confidence level. The coefficient for the lagged Gini on the seigniorage tax rate is 1.316 with a significance level of 0.097. The point estimate implies that a one percentage point cyclical increase of the Gini coefficient (measured on a scale of 0 to 100) is associated with a 1.3 percentage increase in the next period's seigniorage tax rate. Additionally, the R-squared coefficient indicates that 25% of all fluctuations in the monetary policy variable are explained by the lagged values. The predictive power of the regressions significantly drops when the Gini variable is omitted. Panel B in Table 2 presents the estimation results with, for example, just the lagged policy variable; the regression produces an extremely low R-squared.

Additional dummy slope coefficients are added to the regressions. The test for instability around 1979 in the monetary policy equation is rejected indicating that the results of Table 2 are stable across the two periods. Parameter instability is only marginally found when all variables except GDP are dropped. However, the regression is insignificant and has an extremely low R-squared, of about 2%. The results of monetary policy parameter stability of Sims (1980, 1999) and Bernanke and Mihov (1998) are thus reinforced. Note that the regression stability implies that the correlation between the seigniorage tax rate and the lagged Gini must have been lower after 1979 if the seigniorage tax rate became more volatile. Indeed, Table 1 is consistent with this fact.

The four panels of Figure 2 show how the variables respond to a one-standard deviation shock associated with each vectored equation through the impulse response functions that are computed by the generalized method (non-orthogonalized) of Pesaran and Shin (1997). In the first panel, a shock to the monetary equation is plotted. Most of the responses occur in the seigniorage tax rate; the dynamics of the Gini, GDP, and inflation are initially small. The second period shows that output and inflation slightly increase with inequality remaining essentially unchanged. The next panel indicates that monetary policy and output respond, in a significant way, to changes in the shocks associated with the inequality equation. The impulses show that both monetary policy and output fall then to be followed by an increase in the second period. The last two panels show that shocks to the GDP equation and inflation equation affect each other; monetary policy and inequality do not significantly respond.

⁴Below, the model is re-estimated with unemployment in a check of robustness.

Panel A: System Estimation							
	$\Delta \theta_t$	$\Delta qini_t$	Δy_t	$\Delta \pi_t$			
$\Delta \theta_{t-1}$	$-\overline{0.033}_{(0.840)}$	-0.059 (0.191)	$\overline{0.318}_{(0.452)}$	0.558^{*}			
$\Delta gini_{t-1}$	1.316^{**}	-0.519^{**}	1.115 (0.506)	$2.363^{*}_{(0.009)}$			
Δy_{t-1}	-0.043 $_{(0.645)}$	0.036^{**} (0.057)	0.277^{**} (0.088)	$0.513^{*}_{(0.001)}$			
$\Delta \pi_{t-1}$	$-0.177^{*}_{(0.011)}$	$-0.077^{*}_{(0.034)}$	-0.605 (0.140)	$0.622^{*}_{(0.001)}$			
R^2	0.253	0.205	0.366	0.674			
Break(1979)	$\hat{\chi}^2 = \underset{(0.491)}{3.41}$	$\hat{\chi}^2 = \mathop{7.65}_{(0.105)}$	$\hat{\chi}^2 = 16.66^*_{(0.002)}$	$\hat{\chi}^2 = 5.69_{(0.223)}$			
Panel B: Single Equation Estimation							
	$\Delta \theta_t$	$\Delta \theta_t$	$\Delta \theta_t$	$\Delta \theta_t$			
$\Delta \theta_{t-1}$	$-\overline{0.115}_{(0.355)}$	$\overline{0.026}_{(0.849)}$	$-\overline{0.121}_{(0.316)}$	$-\overline{0.168}_{(0.306)}$			
$\Delta gini_{t-1}$	—	$1.801^{*}_{(0.007)}$	—	_			
Δy_{t-1}	—	—	$\underset{(0.462)}{0.069}$	—			
$\Delta \pi_{t-1}$	—	—	—	$-0.263^{*}_{(0.001)}$			
R^2	0.006	0.203	0.020	0.160			
Break(1979)		0		0			

Table 2: GMM Estimation Results (1965-1992)

Notes: Differenced values used; HAC covariances are estimated with a "Newey-West" kernel (Newey and West, 1987) of order one; p-values in parentheses; *Significant at 5%; and **Significant at 10%.



To check the robustness of the results, the regression models are estimated with the unemployment variable. Essentially, the same results are found. Specifically, the coefficient for the lagged Gini on the seigniorage tax rate is 1.446 with a significance level of 0.058. The R-squared coefficient indicates that 29% of all fluctuations in the monetary policy variable are explained by the lagged values. Finally, the test for instability around 1979 in the monetary policy equation is again rejected.

Care should be taken in interpretation of these results. For sure the Federal Reserve emphasizes inflation, output, and labor statistics when discussing policy. Additionally, policy decisions with regard to these variables have changed on an extensive margin since 1979; a graph of the trend rate of money growth shows this. However, this analysis is short-run in nature where the likely consequences of temporary deviations in policy include redistribution of income (*i.e.*, winners and losers). With this in mind, the presented results suggest that monetary policy makers understand the effects on redistribution and thus respond over the business cycle; monetary policy is contemporaneously and inversely related to inequality. Though deviations in inequality feedback into monetary policy, temporary changes in policy have little effect on inequality. Additionally, the policy rule that relates cyclical inequality to policy appears stable but the covariance structure has changed towards policy being more countercyclical. Therefore, the model presented in the next section will allow for these general features.

3 The Model Economy

The model economy is populated by households who are separated into two occupations: workers and capitalists. Workers do not own capital – they merely supply labor to firms. Capitalists, by contrast, can rent capital to firms. This separation, used previously in the growth model by Judd (1985) and Krusell (2002), allows us to highlight how different government policies affect different groups.

For the agents, time evolves in discrete units called periods (which are specified to be one year long in the quantitative results reported later on). A period has two parts in which the economic agents make decisions: the beginning of the period and the end of the period. In the beginning of the period, the monetary authority injects money into the economy by transferring lump-sum cash to the households. At the end of the period, the households, who are in possession of the economy's entire stock of money, make decisions on labor, consumption, investment in capital, and money investment. Note that both groups can save by holding fiat money.

3.1 The Workers

The time spent on transactions-related activities is assumed to be given by the expression:

$$\omega_0 - \omega_1 \left(\frac{m_t^w}{p_t c_t^w}\right)^{\omega_2},$$

where m^w is the current nominal money balance, c_t^w is current real consumption, and p_t is the aggregate price level. Then leisure time in period t is

$$\ell_t^w \equiv T - n_t^w - \omega_0 + \omega_1 \left(\frac{m_t^w}{p_t c_t^w}\right)^{\omega_2}$$

where T is the total time endowment. To insure a solution, the parameters are restricted so that $\omega_2 < 1$ and ω_2 and ω_1 have the same sign. This specification is by Kydland (1991) and Gavin and Kydland (1999).

Workers solve the dynamic programming problem

$$\mathcal{U}^{w}(s_{t}) = \max_{\{c_{t}^{w}, m_{t+1}^{w}, n_{t}^{w}\}} \{ u^{w}(c_{t}^{w}, \ell_{t}^{w}(c_{t}^{w}, m_{t}^{w}, n_{t}^{w})) + E_{t}\beta^{w}\mathcal{U}^{w}(s_{t+1}) \}$$

subject to the budget constraint

$$c_t^w + \frac{m_{t+1}^w}{p_t} \le \frac{m_t^w}{p_t} + (1 - \tau_t)w_t n_t^w + T_t^w,$$

where w_t is the wage rate, n_t^w is the labor supply, T_t^w is the lump-sum real transfer from the government, s_t is a vector of state variables, and E_t is the conditional expectation operator given the current full level of information at time t.

3.2 The Capitalists

Capitalists solve the dynamic programming problem

$$\mathcal{U}^{k}(s_{t}) = \max_{\{c_{t}^{k}, k_{t+1}, m_{t+1}^{k}\}} \left\{ u^{k} \left(c_{t}^{k}, \ell_{t}^{k} (c_{t}^{k}, m_{t}^{k}) \right) + E_{t} \beta^{k} \mathcal{U}^{k}(s_{t+1}) \right\}$$

subject to the budget constraint

$$c_t^k + k_{t+1} + \frac{m_{t+1}^k}{p_t} \le \frac{m_t^k}{p_t} + (1 - \tau_t)w_t \,\bar{n} + \left[(1 - \tau_t)R_t^k + 1 - \delta\right]k_t + T_t^k,$$

where k_t is the amount of capital, R_t^k is the real return to capital, and $\delta \in (0, 1]$ is the depreciation rate.

Leisure time, which is a function of a fixed labor supply choice for the capitalist, \bar{n} , is given by

$$\ell_t^k \equiv T - \bar{n} - \omega_0 + \omega_1 \left(\frac{m_t^k}{p_t c_t^k}\right)^{\omega_2}.$$

Note that the assumption that money facilitates transactions allows the capitalist to hold nonzero amounts of money even though capital rate-of-return dominates money.

3.3 The Firm

The representative firm rents capital and hires labor. The firm produces consumption goods via a neoclassical constant returns to scale production function and chooses $\{k_t, n_t\}$ to maximize:

$$\pi_t = A_t F(k_t, n_t) - w_t n_t - R_t^k k_t.$$

The firm takes as given $\{w_t, R_t^k\}$. In equilibrium, the factors of production are paid their marginal products:

$$A_t F_1(k_t, n_t) = R_t^k, \quad A_t F_2(k_t, n_t) = w_t$$

where $F_1(\cdot) = \partial F(\cdot)/\partial k_t$ and $F_2(\cdot) = \partial F(\cdot)/\partial n_t$. The Cobb-Douglas form is chosen for the production technology because it is consistent with the relative constancy of income shares:

$$Y_t = A_t F\left(k_t, n_t\right) = A_t k_t^{\alpha} n_t^{1-\alpha},$$

where $\alpha \in (0, 1)$ is the share of income that goes to capital. Finally, I assume the log of aggregate technology, denoted $a = \ln(A)$, follows the process:

$$a_{t+1} = \phi a_t + \sigma \varepsilon_{t+1},$$

where ε is independent and N(0, 1).

3.4 The Government and Monetary Authority

The government and monetary authority are assumed to pool all revenue for government consumption and transfers. The level of government consumption is assumed to be a fixed fraction ξ of total output. To keep the focus on monetary policy, the income tax rate is set at this fixed rate: $\tau_t = \bar{\tau} = \xi$. Thus, the government's budget is

$$T_{t}^{k} + T_{t}^{w} + \xi Y_{t} = \frac{m_{t+1} - m_{t}}{p_{t}} + \bar{\tau} \left(w_{t} n_{t} + R_{t}^{k} k_{t} \right).$$

Using the law of motion $m_{t+1} = (1 + \theta_t)m_t$, the public budget constraint is rewritten as

$$T_t^k + T_t^w + \xi Y_t = \theta_t \frac{m_t}{p_t} + \bar{\tau} \left(w_t n_t + R_t^k k_t \right);$$

the term θ_t is the seigniorage tax rate.

Transfers are assumed to be made to the workers as a fraction λ of the total. For any λ , transfers written in stationary form⁵ are:

$$T_t^w = \lambda \left[\frac{\theta_t \hat{m}_t}{(1+\theta_t)\hat{p}_t} \right], \quad T_t^k = (1-\lambda) \left[\frac{\theta_t \hat{m}_t}{(1+\theta_t)\hat{p}_t} \right].$$

The primary purpose of this assumption is to capture the potential distributional effects of inflation that are not captured by the model. For example, the model does not have a private market for loanable funds; λ allows for the replication of the transfer of income that takes place from borrower to lender. Note that the parameter λ is the weight attached to workers by some economic mechanism other than monetary policy.

The monetary policy feedback rule that the seigniorage tax rate is assumed to follow is

$$\theta_{t+1} = \theta_0 + \theta_1 \left[\theta_t - \theta_0 \right] + \theta_2 \left[ineq_t - \theta_3 \right] + \sigma_\theta \varepsilon_{\theta,t+1} \tag{1}$$

where $ineq_t$ is a measure of inequality, ε_{θ} is an exogenous shock that is independently distributed standard normal, θ 's are parameters, and σ_{θ} is a scale variable. Because of the assumption that the monetary authority sets transfers by its choice of seigniorage tax rate, the inequality index, $ineq_t$, is defined on pretax and pre-transfer income, or *earnings*. More specifically, the following definition for computation of $ineq_t$ is used:

$$ineq_t = 1 - 2\left(\frac{w_t n_t^w}{A_t F(k_t, n_t)}\right).$$

For computation of the Gini coefficient, the following definition is used:

$$gini_{t} = 1 - 2\left(\frac{w_{t}n_{t}^{w} + T_{t}^{w}}{A_{t}F(k_{t}, n_{t}) + T_{t}^{w} + T_{t}^{k}}\right),$$

which includes pretax and post-transfer income; this is the *money income* definition used by the U.S. Bureau of the Census.

⁵Since p and m will be growing over time, all nominal variables are deflated by the money stock to obtain a stationary environment. This results in the equilibrium conditions $\hat{m}_{t+1} = \hat{m}_t = 1$, $\hat{p}_t = p_t/m_{t+1}$, $\hat{m}_t^k = m_t^k/m_t$, and $\hat{m}_t^w = 1 - \hat{m}_t^k$.

4 Characterization of the Equilibrium

4.1 The Stationary Recursive Problem

The budget constraints in stationary form are:

$$c_t^w + \frac{1 - \hat{m}_{t+1}^k}{\hat{p}_t} \le \frac{1 - \hat{m}_t^k}{(1 + \theta_t)\hat{p}_t} + (1 - \bar{\tau})w_t n_t + T_t^w$$
$$c_t^k + k_{t+1} + \frac{\hat{m}_{t+1}^k}{\hat{p}_t} \le \frac{\hat{m}_t^k}{(1 + \theta_t)\hat{p}_t} + (1 - \bar{\tau})w_t \bar{n} + [R_t^k(1 - \bar{\tau}) + 1 - \delta] k_t + T_t^k.$$

Given the setup, the worker's, the capitalist's, and the firm's optimal behavior define the Stochastic Euler Equations (SEEs):

$$1 = (1 - \tau_t) w_t \left\{ \frac{(u_1^w(t) + [u_2^w(t)\ell_1^w(t)])}{-u_2^w(t)\ell_3^w(t)} \right\}$$
(2a)

$$1 = E_t \frac{\beta^w \hat{p}_t}{(1+\theta_{t+1})\hat{p}_{t+1}} \left\{ \frac{u_1^w(t+1) + u_2^w(t+1) \left[\ell_1^w(t+1) + (1+\theta_{t+1})\hat{p}_{t+1}\ell_2^w(t+1)\right]}{u_1^w(t) + u_2^w(t)\ell_1^w(t)} \right\}$$
(2b)

$$1 = E_t \beta^k [R_{t+1}^k(1-\bar{\tau}) + 1 - \delta] \left\{ \frac{u_1(t+1) + u_2(t+1)\iota_1(t+1)}{u_1^k(t) + u_2^k(t)\ell_1^k(t)} \right\}$$
(2c)

$$1 = E_t \frac{\beta^k \hat{p}_t}{(1+\theta_{t+1})\hat{p}_{t+1}} \left\{ \frac{u_1^k(t+1) + u_2^k(t+1) \left[\ell_1^k(t+1) + (1+\theta_{t+1})\hat{p}_{t+1}\ell_2^k(t+1)\right]}{u_1^k(t) + u_2^k(t)\ell_1^k(t)} \right\}$$
(2d)

and the equilibrium functions of the current states: $\hat{m}_{t+1}^k = \mathcal{M}(s_t), k_{t+1} = \mathcal{H}(s_t), n_t^w = \mathcal{N}(s_t), \text{ and } \hat{p}_t = \mathcal{P}(s_t), \text{ where } s_t \equiv \{k_t, \hat{m}_t^k, a_t, \theta_t\} \text{ and }$

$$u_1(t) \equiv \frac{\partial u(c_t, \ell_t)}{\partial c_t} \quad u_2(t) \equiv \frac{\partial u(c_t, \ell_t)}{\partial \ell_t} \\ \ell_1(t) \equiv \frac{\partial \ell(c_t, m_t)}{\partial c_t} \quad \ell_2(t) \equiv \frac{\partial \ell(c_t, m_t)}{\partial m_t}.$$

The equilibrium functions and constant returns to scale in production then define the

worker's and the capitalist's consumptions by:

$$\begin{aligned} \mathcal{C}^w(s_t) &= \frac{1 - \hat{m}_t^k}{(1 + \theta_t) \mathcal{P}(s_t)} + (1 - \bar{\tau}) w_t \mathcal{N}(s_t) - \frac{1 - \mathcal{M}(s_t)}{\mathcal{P}(s_t)} + \\ &\lambda \left[\frac{\theta_t \hat{m}_t}{(1 + \theta_t) \mathcal{P}(s_t)} + (\bar{\tau} - \xi) Y_t \right]; \\ \mathcal{C}^k(s_t) &= \frac{\hat{m}_t^k}{(1 + \theta_t) \mathcal{P}(s_t)} + (1 - \bar{\tau}) w_t \bar{n} + \left[R_t^k (1 - \bar{\tau}) + 1 - \delta \right] k_t - \\ &\frac{\mathcal{M}(s_t)}{\mathcal{P}(s_t)} - \mathcal{H}(s_t) + (1 - \lambda) \left[\frac{\theta_t \hat{m}_t}{(1 + \theta_t) \mathcal{P}(s_t)} + (\bar{\tau} - \xi) Y_t \right]. \end{aligned}$$

4.2 The Equilibrium

We are now in a position to define an equilibrium for our economy.

Definition 1 A competitive equilibrium for this economy consists of a savings function $\mathcal{H}(s)$, a labor supply function $\mathcal{N}(s) + \bar{n}$, a money demand function $\mathcal{M}(s)$, consumption functions $\mathcal{C}^w(s)$ and $\mathcal{C}^k(s)$, pricing functions w(s), $R^k(s)$, and $\mathcal{P}(s)$, and policy functions $\theta(s)$, and $T^w + T^k = T(s)$ such that

- (i) $\mathcal{C}^{w}(s)$, $\mathcal{N}(s)$, and $1 \mathcal{M}(s)$ solve the worker's intratemporal condition and the budget constraint for the given prices and policies;
- (ii) $\mathcal{H}(s)$, $\mathcal{C}^{k}(s)$, and $\mathcal{M}(s)$ solve the capitalist's Euler equation and the budget constraint for the given prices and policies;
- (iii) the firm's first-order conditions are satisfied, given prices;
- (iv) the goods market clears:

$$\mathcal{C}^{w}\left(k,\hat{m}^{k},a\right) + \mathcal{C}^{k}\left(k,\hat{m}^{k},a\right) + \mathcal{H}\left(k,\hat{m}^{k},a\right) - (1-\delta)k = (1-\bar{\tau})e^{a}F\left(k,\mathcal{N}(k,\hat{m}^{k},a) + \bar{n}\right);$$

and

(v) the government budget constraint holds:

$$T(s) = \frac{\theta(s)}{[1+\theta(s)]\mathcal{P}(s)} + (\bar{\tau} - \xi) e^a F\left(k, \mathcal{N}(k, \hat{m}^k, a) + \bar{n}\right)$$
$$= \frac{\theta(s)}{[1+\theta(s)]\mathcal{P}(s)}.$$

5 Solution and Calibration Methods

5.1 Perturbation

For computation of the equilibria, the smooth approximation method of Judd (1998) and Gaspard and Judd (1997), which relies on a second-order Taylor series expansion, is applied to the functional equations, the SEEs, that jointly characterize the equilibrium. As in Schmitt-Grohé and Uribe (2002), the scale parameters for the variance of the exogenous shocks, σ and σ_{θ} , are incorporated and used as an argument in differentiation for the expansion. The center for the expansion is the deterministic rest point of the economy. Letting the states of the economy be defined as $s = \{k, \hat{m}^k, a, \theta, \sigma, \sigma_{\theta}\}$, the private agent's policy functions are then approximated, for example, by the expansions:

$$\begin{aligned} \mathcal{H}(s) &= \mathcal{H}^{(0)} + \mathcal{H}_{1}^{(1)}(k - \bar{k}) + \mathcal{H}_{2}^{(1)}(\hat{m}^{k} - \bar{m}^{k}) + \mathcal{H}_{3}^{(1)}(a) + \\ & \mathcal{H}_{4}^{(1)}(\theta - \theta_{0}) + \mathcal{H}_{5}^{(1)}(\sigma) + \mathcal{H}_{6}^{(1)}(\sigma_{\theta}) + \\ & \frac{1}{2}\mathcal{H}_{1}^{(2)}(k - \bar{k})^{2} + \frac{1}{2}\mathcal{H}_{2}^{(2)}(\hat{m}^{k} - \bar{\bar{m}}^{k})^{2} + \ldots + \\ & \mathcal{H}_{1,2}^{(2)}(k - \bar{k})(\hat{m}^{k} - \bar{\bar{m}}^{k}) + \mathcal{H}_{1,3}^{(2)}(k - \bar{k})(a) + \ldots \end{aligned}$$

where $\bar{k} = \mathcal{H}^{(0)} = \mathcal{H}(\bar{k}, \bar{\hat{m}}^k, 0, \theta_0, 0, 0)$, and

$$\mathcal{H}^{(1)} = \left. \frac{\partial \mathcal{H}}{\partial s} \right|_{s=\bar{s}}, \, \mathcal{H}^{(2)} = \left. \frac{\partial^2 \mathcal{H}}{\partial s \, \partial s^{\top}} \right|_{s=\bar{s}}, \dots,$$

with \mathcal{M}, \mathcal{N} , and \mathcal{P} defined likewise.

The SEEs, equations (2a)-(2d), evaluated at \bar{s} are denoted, respectively, as

$$0 = SEE_i^{(0)}(\mathcal{H}^{(0)}, \mathcal{M}^{(0)}, \mathcal{N}^{(0)}, \mathcal{P}^{(0)})|_{i=a,...,d}$$

Taking the derivative n times of equations (2a)-(2d) and evaluating at \bar{s} produces additional equations that the competitive equilibrium must satisfy. These are denoted by

$$0 = SEE_i^{(n)}(\mathcal{H}^{(0)}, \mathcal{M}^{(0)}, \mathcal{N}^{(0)}, \mathcal{P}^{(0)}, \dots, \mathcal{H}^{(n)}, \mathcal{M}^{(n)}, \mathcal{N}^{(n)}, \mathcal{P}^{(n)})|_{i=a,\dots,d}$$

where i = a, ..., d identifies the equation. In practice, the expansion is terminated after the quadratic terms; higher-order coefficients become very small and do not justify the additional computational burden. Thus, the solution, $\{\mathcal{H}^{(i)}, \mathcal{M}^{(i)}, \mathcal{N}^{(i)}, \mathcal{P}^{(i)}\}_{i=0}^2$, can be derived by taking successive derivatives (up to n = 2) and setting any higher-order polynomial coefficients

- for example, $\mathcal{H}^{(3)}$, $\mathcal{M}^{(3)}$, $\mathcal{N}^{(3)}$, and $\mathcal{P}^{(3)}$ - to zero.

This problem has a recursive structure and can be solved by first finding the steady states from the first three equations: $\{\mathcal{H}^{(0)}, \mathcal{M}^{(0)}, \mathcal{P}^{(0)}, \mathcal{P}^{(0)}\}$. Second, the higher-order coefficients, $\{\mathcal{H}^{(i)}, \mathcal{M}^{(i)}, \mathcal{N}^{(i)}, \mathcal{P}^{(i)}\}_{i=1}^{2}$, are found from the remaining equations. Because this second step might yield two solutions, an unstable and stable solution, the parameter search for $\mathcal{H}_{1}^{(1)}$ and $\mathcal{M}_{2}^{(1)}$ is restricted to be in absolute value less than one as suggested by Krusell (2002). Finally, starting from \bar{s} , the time series for the allocations and prices are derived from 8,000 simulated technology shocks; the first 1,000 of each series are dropped to eliminate any transitional dynamics.

Note that the procedure does not impose certainty equivalence in the decision rules. However, it turns out that the first-order terms are zero in σ and σ_{θ} ; as in Schmitt-Grohé and Uribe (2002), uncertainty is found to have at most second-order effects on decision rules. Furthermore, the cross terms are also zero. Even though the impact of uncertainty is second-order, it turns out to be quantitatively important.

5.2 Calibration

To conduct the quantitative analysis, the functional forms for utility must be selected. Since there is no trend in hours worked in the data, but there is a trend in wages, the momentary utility function is chosen for workers and capitalists from the family of constant relative risk aversion:

$$u^{w}(c_{t}^{w},\ell_{t}^{w}) = \frac{\left[(c_{t}^{w})^{\mu}(\ell_{t}^{w})^{1-\mu}\right]^{1-\gamma}}{1-\gamma}, \quad u^{k}(c_{t}^{k},\ell_{t}^{k}) = \frac{\left[\left(c_{t}^{k}\right)^{\mu}(\ell_{t}^{k})^{1-\mu}\right]^{1-\gamma}}{1-\gamma}$$

The parameter γ is the Arrow-Pratt coefficient of relative risk aversion; as a benchmark $\gamma = 2$ is selected (baseline model). The parameter μ is a relative share parameter selected to match aggregate labor supply equal to one-third of the time endowment. This results in a value of $\mu = 0.33$.

For the real economy, the following vector of parameters is to be calibrated: $\Theta_1 \equiv [\alpha, \beta, \delta, \phi, \sigma, \xi, \lambda]$. I choose $\alpha = 0.36$, which roughly matches the share of capital income in output for the United States since the Second World War. The average discount rate β is fixed at a value compatible with a yearly psychological rate of 3%. The depreciation rate is set at $\delta = 0.0435$, which obtains a steady-state investment/GDP ratio of 0.15. The government's share parameter ξ is set at 21.4%, the average U.S. share of total government consumption (18%) plus net interest expenses (3.4%) in output since 1960. The parameters for the technology process a are set by $\phi = 0.50$ and $\sigma = 0.0304$, the annual values most commonly found in the literature. For now, the transfer weight λ is arbitrarily set to 0.50;

different values for this parameter are considered.

Preferences:	$\beta = 0.97, \gamma = 2.0, \mu = 0.33$
Technology:	$\alpha = 0.36, \phi = 0.85, \sigma = 0.0304$
	$\delta = 0.0435$
Transaction Costs:	$\omega_0 = -0.020043, \omega_1 = -0.0136$
	$\omega_2 = -1.0, \bar{n} = 0.33$
Government:	$\xi = 0.214, \lambda = 0.50$
Monetary Auth.:	$\theta_0 = 0.06, \theta_1 = -0.115, \theta_2 = 0.0, \theta_3 = 0.0$
	$\sigma_{\theta} = 0.00912$

 Table 3: Parameter Values for the Baseline Model

For the calibration of the transaction cost variables, $\Theta_2 \equiv [\omega_0, \omega_1, \omega_2, \bar{n}]$, Gavin and Kydland (1999) are followed by first setting $\omega_2 = -1.0$ and $\omega_1 = -0.0136$. This sets the interest rate elasticity equal to -0.50 and the real interest rate equal to about 9% per year (or a net rate of 5.25%). Next, for capitalists, the labor supply parameter is fixed to equal the worker's average hours; $\bar{n} = 0.33$. The parameter ω_0 is set so that $\bar{\ell}^w + \bar{n} = T = 1$ at the economy's deterministic rest point. This amounts to setting $\omega_0 = \omega_1 [\bar{m}^w/(\bar{p} \cdot \bar{c}^w)]^{\omega_2}$. The calibration of the monetary policy rule, $\Theta_3 \equiv [\theta_0, \theta_1, \theta_2, \theta_3, \sigma_{\theta}, \epsilon]$, follows the estimation results in Table 2 for when monetary policy follows a simple univariate AR process. Specifically, $\theta_0 = 0.06$, $\theta_1 = -0.115$, $\theta_2 = 0.0$, $\theta_3 = 0$, and $\sigma_{\theta} = 0.00912$; this is to be the initial baseline model. The results for the calibrations are in Table 3.

6 Results

A wide range of experiments are conducted by altering the values for the policy rule parameters and the parameter for transfers. Changes in the policy rule are intended to capture deviations from the mean growth of money and the variance of the shock to the policy rule. Changes in the distribution of transfers allows for the distributional effects of policy to be altered.

6.1 Baseline Economy

The simulation results for the baseline economy are presented in the first panel of Table 4. As expected, the Gini coefficient is inversely related to λ ; the workers are taxed relatively less for high λ . The lower taxation causes the workers to increase their money holdings and decrease work effort. The increases in their transfer income also makes labor effort more elastic and thereby increasing the variance of labor effort. As a result, output decreases and becomes more volatile. In terms of welfare, the total effects of a fall in λ are that average utility for the worker falls and the average variance of utility for the worker increases. Thus, a decrease in λ represents a transfer of welfare from the worker to the capitalist.

The baseline model is unable to replicate the observed cyclical properties of the U.S. economy with respect to inequality and monetary policy. When $\lambda = 0.50$, the simulated contemporaneous correlation of the seigniorage tax rate and the Gini coefficient is $corr(\theta_t, gini_t) = -27.5\%$. An increase in θ would increase the income of the worker, thereby decreasing income inequality. However, the correlation between the seigniorage tax rate and the lagged Gini is extremely small at $corr(\theta_t, gini_{t-1}) = 3.1\%$. Because the seigniorage tax has little persistence, lagged Gini coefficients will be uncorrelated with monetary policy. These results are robust to both increases and decreases in λ .

	$ heta_t\%$	$gini_t\%$	y_t	$n_t\%$	$\hat{m}_t^k\%$	$\frac{\frac{c_t^w}{C_t}}{\frac{c_t^w}{C_t}}$	$u^w(t)$	$u^k(t)$	$\mathit{infla}_t\%$
		Ba	$aseline: \theta$	$_0 = 0.06$	$\theta_1 = -0$	$.115, \theta_2 =$	$= 0.0, \ \theta_3 =$	= 0.0	
$\lambda = 1.00$	$6.000 \\ (0.815)$	$\underset{(0.431)}{35.209}$	30.457 (2.409)	$\underset{(0.318)}{30.705}$	$\begin{array}{c} 59.195 \\ \scriptscriptstyle (0.739) \end{array}$	$\underset{(62.463)}{41.264}$	-1.816 (1.951)	-1.656 (0.939)	$\underset{(3.244)}{6.046}$
$\lambda = 0.75$	$\substack{6.000\\(0.815)}$	$\underset{(0.341)}{35.913}$	$\underset{(2.401)}{31.269}$	$\underset{(0.307)}{31.227}$	$\substack{59.753 \\ (0.721)}$	$\underset{(61.788)}{40.613}$	-1.830 (1.962)	-1.645 $_{(0.935)}$	$\underset{(3.241)}{6.046}$
$\lambda = 0.50$	$6.000 \\ (0.815)$	$\underset{(0.288)}{36.619}$	$\underset{(2.394)}{32.091}$	$\underset{(0.299)}{31.760}$	$\underset{(0.714)}{60.310}$	$\underset{(61.129)}{39.959}$	-1.844 (1.981)	-1.635 (0.932)	$\substack{6.046 \\ (3.265)}$
$\lambda = 0.25$	$6.000 \\ (0.815)$	$\underset{(0.293)}{37.327}$	$\underset{(2.386)}{32.922}$	$\underset{(0.293)}{32.304}$	$\underset{(0.716)}{60.868}$	$\underset{(60.486)}{39.303}$	-1.859 (2.010)	-1.625 (0.930)	$\substack{6.047 \\ (3.316)}$
$\lambda = 0.00$	6.000 (0.815)	$\underset{(0.352)}{38.037}$	$\underset{(2.379)}{33.763}$	$\underset{(0.289)}{32.858}$	$\underset{(0.729)}{61.425}$	$\underset{(59.861)}{38.645}$	-1.875 (2.049)	-1.614 (0.928)	$\underset{(3.390)}{6.049}$
		G	$ini: \theta_0 =$	$= 0.06, \theta_1$	= 0.026,	$\theta_2 = 1.8$	$01, \theta_3 = \overline{i}$	\overline{neq}	
$\lambda = 1.00$	$\underset{(0.956)}{6.033}$	$\underset{(0.533)}{35.206}$	$30.446 \\ (2.445)$	$\underset{(0.349)}{30.698}$	$\underset{(0.785)}{59.206}$	$\underset{(59.789)}{41.267}$	-1.816 (1.768)	-1.656 (1.001)	$\underset{(4.237)}{6.109}$
$\lambda = 0.75$	$\substack{6.025 \\ (0.953)}$	$\underset{(0.410)}{35.912}$	${31.262} \\ {\scriptstyle (2.425)}$	$\underset{(0.327)}{31.223}$	$\substack{59.767 \\ (0.741)}$	$\underset{(60.511)}{40.614}$	-1.830 (1.865)	-1.645 $_{(0.961)}$	$\underset{(3.985)}{6.093}$
$\lambda = 0.50$	$\substack{6.019 \\ (0.953)}$	$\underset{(0.319)}{36.619}$	32.087 (2.406)	$\underset{(0.310)}{31.757}$	$\underset{(0.719)}{60.325}$	$\underset{(60.864)}{39.959}$	-1.844 (1.958)	-1.635 (0.938)	$\underset{(3.831)}{6.082}$
$\lambda = 0.25$	$\substack{6.014 \\ (0.955)}$	$\underset{(0.292)}{37.320}$	$\underset{(2.389)}{32.921}$	$\underset{(0.297)}{32.303}$	$\underset{(0.717)}{60.882}$	$\underset{(60.980)}{39.302}$	-1.859 (2.048)	-1.625 (0.923)	$\substack{6.074 \\ (3.749)}$
$\lambda = 0.00$	$\substack{6.010 \\ (0.958)}$	$\underset{(0.348)}{38.039}$	$\underset{(2.374)}{33.764}$	$\underset{(0.288)}{32.859}$	$\underset{(0.735)}{61.438}$	$\underset{(60.932)}{38.643}$	-1.875 (2.136)	-1.614 (0.912)	$\underset{(3.728)}{6.069}$
		(Gini: $\lambda =$	$0.50, \theta_1$	= 0.026,	$\theta_2 = 1.80$	$01, \theta_3 = \overline{i}$	\overline{neq}	
$\theta_0 = 0.08$	$\underset{(0.922)}{8.016}$	$\underset{(0.280)}{36.613}$	$\underset{(2.378)}{31.801}$	$\underset{(0.279)}{31.572}$	$\underset{(0.710)}{60.383}$	$\underset{(61.263)}{39.930}$	-1.843 (1.977)	-1.636 (0.944)	$8.079 \\ (3.860)$
$\theta_0 = 0.07$	$\substack{7.017 \\ (0.936)}$	$\underset{(0.299)}{36.615}$	$\underset{(2.391)}{31.940}$	$\underset{(0.293)}{31.662}$	$\substack{60.355\\(0.715)}$	$\underset{(61.082)}{39.944}$	-1.844 (1.968)	-1.636 (0.941)	$7.080 \\ (3.848)$
$\theta_0 = 0.05$	$\underset{(0.971)}{5.021}$	$\underset{(0.342)}{36.626}$	$\underset{(2.423)}{32.243}$	$\underset{(0.328)}{31.859}$	$\underset{(0.721)}{60.295}$	$\underset{(60.600)}{39.973}$	-1.845 (1.948)	-1.634 (0.934)	$\underset{(3.806)}{5.083}$
$\theta_0 = 0.04$	$\underset{(0.993)}{4.023}$	$\underset{(0.369)}{36.637}$	$\underset{(2.442)}{32.410}$	$\underset{(0.349)}{31.968}$	$\underset{(0.720)}{60.265}$	$\underset{(60.276)}{39.988}$	-1.846 (1.937)	-1.634 (0.930)	4.084 (3.772)
		Gini:	$\lambda = 0.50$	$\theta_0 = 0.0$	$\theta_{0}, \theta_{1} = 0$	$0.026, \theta_2 =$	$= 1.801, \theta$	$\theta_3 = \overline{ineq}$	
$4\sigma_{\theta}$	6.004 (1.994)	$\underset{(0.366)}{36.619}$	$\underset{(2.408)}{32.099}$	$\underset{(0.318)}{31.765}$	$\underset{(0.722)}{60.319}$	$\underset{(60.831)}{39.961}$	-1.844 (1.962)	-1.635 (0.943)	$\substack{6.068 \\ (4.484)}$
$2\sigma_{\theta}$	$\substack{6.015 \\ (1.281)}$	$\underset{(0.330)}{36.619}$	$\underset{(2.407)}{32.090}$	$\underset{(0.312)}{31.759}$	$\underset{(0.719)}{60.324}$	$\underset{(60.858)}{39.959}$	-1.844 (1.959)	-1.635 (0.939)	$\substack{6.078 \\ (4.002)}$
$\frac{1}{2}\sigma_{\theta}$	$\substack{6.021\\(0.681)}$	$\underset{(0.313)}{36.619}$	$\underset{(2.406)}{32.085}$	$\underset{(0.308)}{31.756}$	$\underset{(0.719)}{60.326}$	$\underset{(60.867)}{39.958}$	-1.844 (1.958)	-1.635 (0.938)	$\underset{(3.714)}{6.084}$
$\frac{1}{4}\sigma_{\theta}$	$\substack{6.022\\(0.592)}$	$\underset{(0.311)}{36.619}$	$\underset{(2.405)}{32.085}$	$\underset{(0.308)}{31.756}$	$\underset{(0.719)}{60.326}$	$\underset{(60.866)}{39.958}$	-1.844 (1.958)	-1.635 (0.938)	$\begin{array}{c} 6.084 \\ (3.679) \end{array}$
<i>Gini</i> : $\theta_1 = 0.026, \ \theta_2 = 1.801, \ \theta_3 = \overline{ineq}$									
$\begin{array}{l} \lambda = 1 \\ \theta_0 = .03 \end{array}$	$\underset{(1.032)}{3.043}$	$\underset{(0.738)}{35.780}$	$\underset{(2.511)}{31.573}$	$31.423 \\ (0.422)$	59.539 (0.796)	40.812 (57.333)	-1.829 (1.744)	-1.646 (1.008)	3.117 (4.199)

 Table 4: Simulation Results for Baseline and Gini-Based Feedback Rule

Note: % Standard deviations in parentheses.

6.2 Gini-Based Feedback Rule Economy

The monetary policy rule will be a function of earnings inequality when $\theta_1 \neq 0$. To include this feature, equation (1) is calibrated to the estimation results of Table 2; this results in $\theta_1 = 0.026, \theta_2 = 1.801$, and $\theta_3 = \overline{ineq}$. There are several features of the model.

First, the second panel of Table 4 indicates an inverse relationship between income inequality and the mean growth of the money supply with respect to changes in λ . Decreasing λ causes total income of the worker to fall and thus encourages labor effort and increases labor earnings. Because earnings inequality falls (as opposed to income inequality), the mean seigniorage tax rate decreases. Though aggregate output increases, decreases in λ are unambiguously welfare-reducing for the worker; the worker's average mean and variance of utility decrease and increase, respectively. Interestingly, having monetary policy as a function of inequality can produce welfare increases for the agent workers. Specifically, the variance of the worker's lifetime utility is lower in the Gini-based economy relative to the baseline economy when $\lambda > 0.25$. Apparently, making monetary policy a function of inequality allows the worker to smooth income shocks. Note that as a result of the smoother income, labor effort becomes more elastic thereby increasing the variance of labor effort and, hence, increasing the variance of output and of the monetary policy variable.

Second, Figure 3 shows that the Gini-based model is able to replicate several features of the economy when the monetary policy feedback rule is endogenous to earnings inequality. Specifically, Figure 3 indicates that an increase in earnings inequality will increase the rate of money growth, thus increasing transfers. As transfers increase, income inequality falls, giving a negative correlation between the Gini coefficient and seigniorage. Because there is persistence in earnings inequality, an increase in income inequality will imply increases in the next period's seigniorage tax rate – the lagged Gini coefficient positively affects current monetary policy.⁶ The relevant correlations are: $corr(\theta_t, gini_t) = -23.71\%$ and $corr(\theta_t, gini_{t-1}) = 54.82\%$.

Third, as depicted in the first panel of Figure 4, the impulse responses from a onestandard error shock to the monetary policy rule equation are contained mainly in the policy variable; there is little variation in output and inequality. Additionally, the second panel indicates that monetary policy and inequality are insensitive to technology shocks. Because the dynamic responses implied by the model are similar to the data, the Gini-based feedback model appears to be consistent with the empirical facts presented in Section 2.

Fourth, the third of panel Table 4 shows that the Gini coefficient appears rather insen-

⁶This relationship is found to hold also for $\lambda > 0.25$.



Figure 3: Comparison of Seigniorage Rate and Gini for Gini-Based Monetary Economy (Note: $\lambda = 0.50$)

sitive to exogenous changes in the seigniorage rate; the Gini only increases from 36.47% to 36.61% for a 4 basis point increase in θ_0 . However, other aggregates appear sensitive to changes in the rate of growth of money. As the seigniorage rate is increased, the distortions to capital accumulation increase causing the demand for investment and labor to fall. Additionally, workers attempt to economize on their money holdings by selling to the capitalist; the fraction of the worker's consumption to total consumption falls. The decreased labor effort and decreased money holdings for the workers have two effects on utility; increased leisure increases mean utility but the decreased savings increases the variance of utility due to the lack of smoothing opportunities. Therefore, exogenous and permanent increases in the seigniorage rate are welfare-ambiguous; the mean of lifetime utility increases but the variance of lifetime utility increases for the workers. For the capitalists, exogenous and permanent increases in the seigniorage rate are welfare-decreasing.



Figure 4: Impulse Response Functions for Gini-Based Monetary Economy (Note: $\lambda = 0.50$)

Finally, the fourth panel of Table 4 indicates that the mean levels for the economy's aggregates are insensitive to changes in the variance of the monetary policy shock σ_{θ} . The only noticeable effect is an increase in the variance of the seigniorage rate and hence the variance of the inflation rate.

6.3 Correlation Structure of the Gini-Based Economy

Figure 5 shows that increases in the seigniorage revenues devoted to the workers decrease the correlation between the seigniorage rate and output as well as the correlation between the seigniorage rate and the lag of the Gini coefficient. Because these correlations diverge in opposite directions from changes in σ_{θ} , increases in λ followed by a fall in θ_0 can replicate the stylized facts of the U.S. economy since 1979. That is, increases in λ decrease both correlations of seigniorage but increase the mean rate of money creation. Simultaneously decreasing θ_0 preserves the correlations while decreasing the mean rate of money growth; Figure 5 indicates that the relevant correlations are insensitive to changes in θ_0 .



Figure 5: Simulated Correlations for Gini-Based Feedback Rule

In order to provide further intuition of what could have happened to monetary policy after 1979, the last panel of Table 4 presents the simulation results of a Gini-based economy where $\lambda = 1$ and $\theta_0 = 0.03$. In this economy, compared to the case where $\lambda = 0.50$ and $\theta_0 = 0.06$, the relatively low rate of money creation combined with the increased λ combine to alter the covariance structure of monetary policy by changing the distribution of inequality. The correlations for when $\lambda = 0.50$ and $\theta_0 = 0.06$ are: $corr(\theta_t, y_t) = -27.15\%$ and $corr(\theta_t, gini_{t-1}) = 54.82\%$. In contrast, the correlations of θ_t when $\lambda = 1$ and $\theta_0 = 0.03$ are considerably lower: $corr(\theta_t, y_t) = -36.89\%$ and $corr(\theta_t, gini_{t-1}) = 9.61\%$. Therefore, the alteration in λ followed subsequently by a fall in θ_0 can replicate the increased countercyclicity

of monetary policy while preserving the stability of the monetary policy rule.

7 Conclusion

This paper has shown that incorporation of a Gini-based monetary feedback rule is generally compatible with several features of the U.S. economy. Specifically, Gini-based feedback rules replicate the relationship between inequality and the seigniorage rate; the lagged Gini coefficient positively affects the current level of monetary policy. For $\lambda > 0.25$, making policy a function of inequality generates welfare benefits to the worker by consumption smoothing. The costs, that get passed through to the capital owners, are from a more destabilized economy.

Does this result suggest that the Federal Reserve set policy to increase the well-being of the poor? At first, the answer appears unclear because the Gini coefficient is only a measure of the poor's share of total income and income of the poor is derived mainly from wages (Eckstein and Nagypál, 2004). The Gini coefficient could be a proxy for the relative performance of the labor market which policy makers have shown an interest in (Bernanke, 2003). Thus, the FED could be setting policy based on labor market conditions and not the welfare of the poor. However, employment rates in Blue Collar occupations, who are considered the working poor, are by far the highest; Eckstein and Nagypál (2004) report rates above 50%. Thus, it follows that when the Federal Reserve makes policy to affect the relative performance of the labor market, they are in effect setting policy for an average worker who is poor.

A second main result found is that increases in the fraction of revenues received by the workers from seigniorage, λ , can replicate the increased countercyclicity of monetary policy while preserving the stability of the monetary policy rule. This result is interesting because there are a number of occurred changes in the economy – other than monetary policy – that could alter the distributional effects of inflation. An example would include the alterations in the tax code that have been found by Heer and Süssmuth (2003) to empirically change the effects of tax bracket creep. Additionally, this result adds to the critique of the theory, presented in Romer and Romer (2002), that policy makers relearned countercyclical policy in the 1980's. Instead, it has been shown that intensive monetary policy changes can alter covariance structure of monetary policy. The search for likely changes in λ is a direction for future research.

The analysis could also be extended to a political-economic model to determine if theory would predict a destabilizing policy that increases the welfare of the poor. In fact, there are examples in the literature that model inflation as the result of political economic power. For example, Dolmas, Huffman, and Wynne (2000) and Albanesi (2000) suggest political conflicts can be a reason for inflation. In both studies, the results are steady-state implications under different political powers of the agents; they do not consider policy implications over the business cycle. The dynamic political-economic models of fiscal policy found in Krusell (2002) and Fowler and Young (2005) would be a guide for the analysis.

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