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The Acquisition of Skills Over the Life-Cycle

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Abstract

The cyclical behavior of the acquisition of skills over the life-cycle is investigated. The OLG model employed includes the human capital production sector of Heckman (1976) that has two possible responses in skill acquisition to a productivity shock; a substitution and an income effect. The calibrated model predicts, for all age groups, that the substitution effect dominates the income effect implying opportunity-cost considerations tend to make schooling countercyclical. However, the data on college enrollments suggests that the ability-to-pay consideration, or the income effect, is more important for the very young since enrollments for the recently graduated from high-school are procyclical. By making human capital acquisition shocks positively correlated with the TFP shock, the income effect of the young is increased thereby replicating the observed data.

Key words: Human Capital; OLG; Perturbation

JEL category: J24; J31; E24

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1 Introduction

The cyclical properties of human capital acquisition has received considerable attention recently. For example, Perli and Sakellaris (2003) find that aggregate college enrollments are countercyclical. In a quantitative study, DeJong and Ingram (2001) confirm that human capital acquisition is countercyclical in an infinitely lived representative agent model. An intuitive explanation can easily be found; a contraction in the economy is a time when wages, the opportunity costs of obtaining skills, are intertemporaly low causing a substitution towards skill acquisition activities. The purpose of this study is to extend the analysis of DeJong and Ingram (2001) so that a quantitative theory on the acquisition of skill over the life-cycle may be developed. More specifically, this paper addresses the question of whether the cyclical demand for skills responds to aggregate production shocks over the life-cycle, and if so, in what manner.

Besides opportunity-costs considerations, the ability-to-pay may also be an important motive for the acquisition of skills. In an economy, the young have relatively less income and longer life-cycles than other age cohorts implying, possibly, different elasticities of demand for skill acquisition. In this case, an income effect, that would increase the supply of labor effort in market activities during contractions, might arise, offsetting the substitution effect. Indeed, by extending the analysis of Reder (1955), Young (2003) has found that the wage premium for a college education, or the *skill premium*, is essentially uncorrelated with income.[1] This lack of systematic comovement between the return to skills and aggregate activity is evidence that the substitution effect is not dominant in the data; that is, the income effect does matter.

The class of model employed for the study is of stochastic production, human capital accumulation, and overlapping generations (OLG). Stochastics enter the model by an autoregressive process that shifts total factor productivity. Human capital, and thus skill

[1] This evidence is also found in Lindquist (2003).

acquisition, is introduced via a human capital transformation function that has been studied by Heckman (1978) and estimated by Heckman, Lochner, and Taber (1998). The OLG structure allows for the replication of heterogeneity in households with respect to their age and, thus, replication of different elasticities of demand for skills.[2] Due to the extreme agent heterogeneity and the additional saving decision (human capital), iterative solution methods will be computationally burdensome, however. For this reason, a perturbation method is employed on the equations that characterize the solution to dynamic programming problems of the agents. This method relies on higher order expansions (greater than first-order) and has been shown by Judd and Gaspar (1997) to be an efficient algorithm in economies with a large number of states.[3]

There are several other reasons that make the study of the responses in the demand for skills to cyclical fluctuations important. First, if the business cycle persistently affects the acquisition of skills of the young, then government policies that insure the young's ability to acquire skills may be appropriate. That is, because the young cannot naturally insure against business cycles during their formative years, insurance for the young's acquisition of skills may be justified in the presence of long-lasting income effects. Second, it is important to know how the cyclical properties of the wage premium are generated. Knowing the response of the demand for skills may help us better explain the responses (or lack of response) of the wage premium to cyclical fluctuations.

The paper is structured in two parts. In the first part, the empirical relationship between skill acquisition and income is documented. Using aggregate enrollment data from the Department of Education and a conditional linear feedback model, enrollments are found to be strongly countercyclical. Alternatively, the disaggregated data on college enrollments

[2] Gomme, et. al. (2004) use a OLG framework to study the volatility of hours over the lifecycle, so it is apparent that the profession sees some value in studying life cycle business cycle models.

[3] A recent conference sponsored by the *Journal of Economic Dynamics and Control* focused on solving heterogeneous-agent economies had a disproportionate number of papers using perturbation methods to solve the models. Our paper could be considered a companion to any of those, focusing the method on a different type of heterogeneous-agent economy.

suggests that the ability-to-pay consideration, or the income effect, is more important for the very young since enrollments for the recently graduated from high-school are procyclical. In this case, an increase in income implies an increase in enrollments for the recently graduated.

In the second part of the paper, the quantitative results of the modeling are presented. In the benchmark model, the relationship between skill acquisition and the business cycle is explored. Even though the relationship between skill acquisition and the business cycle does not resemble the observed pattern for the U.S. economy, this economy is to act as a benchmark against which to compare the results. Specifically, for all age groups, when a boom in productivity occurs, the intertemporally-high wage induces all households to shift hours to the now more productive goods sector, implying a dominant substitution effect. The correlation between skill acquisition hours and output is a decreasing function with age, reflecting the fact that agents close to retirement have little incentive to substitute current consumption for future consumption, both because they have little time to recoup this loss and because they already have high skills.

DeJong and Ingram (2001) find that the total factor productivity of the educational sector is nearly perfectly correlated with TFP in the goods-producing sector. Modification of the benchmark model in incorporate this correlation gives an ability-to-pay feature that allows the life-cycle model to replicate the procyclical behavior of skill acquisition for the very young; the effect of making the two productivities positively correlated is to make the return to the skill acquisition more procyclical, leading to increased hours studying for the young. For the middle-aged, the counter-cyclical behavior of skill acquisition is reduced but not enough to make their hours procyclical.

The rest of the paper is organized as follows. Section 2 presents some relevant empirical facts. Section 3 lays out the theoretical model. Section 4 presents the calibrations and discusses the solution method. Section 5 presents the model's results. Finally, Section 6 concludes by comparing the quantitative model to the empirics.

2 Empirics

This section presents some basic facts about the business cycle behavior for the acquisition of skills. The data are from the IMF's International Financial Statistics (IFS) database and the Department of Education's Integrated Postsecondary Education Data System (IPEDS). From the IFS database GDP, population, and the CPI for the years 1975 to 2000 are identified. The IPEDS data set has total fall enrollments for all degree-granting institutions from 1975 to 2000. This data set also offers disaggregated enrollments in, for example, 4 and 2 year institutions, undergraduate and graduate programs, and by recent high-school graduates (within 12 months).

The three panels of Figure 1 display the historical pattern of various per capita enrollment series. The first panel illustrates that per capita total enrollments, both full and part time, have been growing over time. The second and third panels in Figure 1 shows that most of the variability in enrollments are from 2-year undergraduate programs. Graduate enrollment seems not to be affected over the business cycle and are, hence, acyclical. Thus, most of the variations in skill acquisition, as measured by college enrollments, are from students with limited college experience. The two panels of Figure 2 illustrate the cyclical components[4] of college enrollments (less recently graduated high school seniors), enrollments of recently graduated high-school seniors, and per capita real GDP. The cyclical component of college enrollment displays strong countercyclicity with a correlation coefficient with GDP of -0.57. In the second panel, enrollment of recently graduated high-school seniors displays procyclicity with a correlation coefficient with GDP of 0.205.

In general, there are two main economic effects that are used to describe the forces that drive the cyclicity of enrollments; an income and substitution effect. The income effect would discourage nonmarket activities during economic contractions; the lack of income encourages

[4] The cyclical components are measured by the Hodrick-Prescott filter where the smoothing parameter has been set at 6.5.

market activities for the replacement of the lost income. Alternatively, the substitution effect would encourage the reallocation of time to skill acquisition activities and out of market activities; this is due to the intertemporaly low wages. Economic reasoning and Figure 2 suggests that the income effect is more important for the young; the young have relatively less income to insure against negative income shocks and thus would seek to replace lost income in market activities. Alternatively, the middle-aged have accumulated savings as self insurance and thus would not, relatively, seek to replace lost income in market activities which have intertemporarly low returns.

To more formally test the cyclical properties of enrollments, a conditional linear feedback model (LFM) is estimated for the process that governs enrollments. The LFM is defined by

$$E(y_{i,t}|y_{i,t-1}, \mathbf{x}_t) = \gamma_i y_{i,t-1} + \exp(\mathbf{x}_t \boldsymbol{\beta}_i),$$

where $y_{i,t}$ is time t enrollments of type i student (count data) and $\mathbf{x}_t$ is a vector of explanators.[5] The quasi-differenced estimator of Woodridge (1997) for the LFM is found by Generalized Method of Moments (GMM) that is to minimize the squared metric of the sum of residuals, given by

$$r_i = (y_{i,t} - \gamma_i y_{i,t-1}) \frac{\exp(\mathbf{x}_{t-1} \boldsymbol{\beta}_i)}{\exp(\mathbf{x}_t \boldsymbol{\beta}_i)} - (y_{i,t-1} - \gamma_i y_{i,t-2}),$$

with instruments given by the lags of $\mathbf{x}_t$. Additionally, the GMM weighting matrix is defined by the Newey-West kernel (Newey and West, 1987) set at a lag length of 2. The independent variables used for $\mathbf{x}_t$ are a constant (C), the log of real per capita gross domestic product (GDP), the log of population (POP), and a time trend. Given the quasi-differenced setup, the time trend becomes the constant. The dependent variables are college enrollments less recently graduated high school seniors (ENROL) and college enrollments of recently graduated

[5] See Blundell, Griffith, and Windmeijer (2002) for discussion on the LFM.

high-school seniors (HI2COL).

Table 1 presents the estimation results for the LFM. Table 1 shows that the acquisition of skills, as measured by per-capita college enrollments, are countercyclical; the coefficient for GDP is significant, negative, and implies that a one percent increase in GDP decreases enrollments by 42.33 percent. Alternatively, the acquisition of skills for the young (assuming recently graduate high-school seniors are the young) display procyclicity; the coefficient for GDP is significant, positive, and implies that a one percent increase in GDP increases enrollment of the recently graduated by 71.17 percent.

These results appear odd in that the coefficient for population is not significant. However, the GDP series is in per capita terms. Thus, changes in population may indirectly affect enrollments by the combination of its direct parameter estimate and GDP's parameter estimate. Also note that the F statistic for the HI2COL equation is low; the F statistic is insignificant at 5 percent. However, the Sargan test of the joint validity of the instrument is not rejected when computed from the sum of the objective function.

3 The Model

The model has three sectors: the household sector, the production sector, and the skill acquisition sector. There are two types of agents that make economic decisions: households and firms. We assume that each household operates its own skill-acquisition technology.

3.1 The Households

Each period a large number of agents are born with identical preferences and identical initial capital stocks. Agents from generation t live for I periods and then die. At any point in time there is a set of agents indexed by $\tau \in \mathcal{I} = \{0, 1, 2, \ldots, I-1\}$. The time t problem of the agent born in period t is given by

$$\max E_t \left\{ \sum_{\tau=0}^{I-1} \beta^\tau \Psi_\tau u(c_{t+\tau}^t, n_{1,t+\tau}^t, n_{2,t+\tau}^t) \right\},$$

where c is consumption, n_1 is time devoted as labor, and n_2 is time devoted to human capital accumulation or skill acquisition. Additionally, $\Psi_\tau = \prod_{i=0}^{\tau} \psi_i$ denotes the unconditional probability of surviving up to age τ with each ψ_τ representing the conditional probability of surviving from age $\tau - 1$ to τ. The budget constraints of a typical consumer born at time t at any time $t+\tau$ where $\tau \in \mathcal{I}$ is

$$\begin{aligned} c_{t+\tau}^t + k_{t+\tau+1}^t &\leq (1 + r_{t+\tau} - \delta_k) k_{t+\tau}^t + w_{t+\tau} h_{t+\tau}^t n_{1,t+\tau}^t \\ h_{t+\tau+1}^t &\leq q_{t+\tau}^t + (1 - \delta_h) h_{t+\tau}^t, \end{aligned}$$

where w is the labor wage rate, r is the return to physical capital, δ's are the depreciation rates, and q is a human capital production function operated by each agent. Additionally, there are no private annuity markets.

More specifically, human capital is assumed to be produced by the transformation function:

$$q_{t+\tau}^t = \exp(a_{t+\tau}^t) \left(h_{t+\tau}^t\right)^{\theta_1} \left(n_{2,t+\tau}^t\right)^{\theta_2},$$

for all t and $\tau \in \mathcal{I}$, and where $\exp(a_{t+\tau}^t)$ is an exogenous shock that shifts the efficiency of the human capital technology. The marginal products with respect to human capital and hours are denoted, respectively, as:

$$\begin{aligned} q_{h,t+\tau}^t &= \exp(a_{t+\tau}^t) \theta_1 \left(h_{t+\tau}^t\right)^{\theta_1 - 1} \left(n_{2,t+\tau}^t\right)^{\theta_2} \\ q_{n,t+\tau}^t &= \exp(a_{t+\tau}^t) \theta_2 \left(h_{t+\tau}^t\right)^{\theta_1} \left(n_{2,t+\tau}^t\right)^{\theta_2 - 1}. \end{aligned}$$

The benefit of using these functional forms is that they imply a gross aggregate return similar to the one estimated and calibrated by Heckman (1978) and Heckman, Lochner, and Taber (1998), for example. Additionally, the input of goods into the production of human capital is ignored. This assumption facilitates equilibrium computations and is not restrictive since goods could always be introduced and then solved out as a function of n_2, thereby reinterpreting n_2 as a goods-time investment composite (Heckman et al., 1998)

3.2 The Firms

Firms combine capital with labor services to produce goods according to constant-returns-to-scale production functions. All functions are assumed to be concave, increasing, and twice continuously differentiable in both labor and capital. The output in this economy is uncertain due to an aggregate random shock to total factor productivity, which is denoted Z. Specifically, the aggregate output from a firm is produced according to a Cobb-Douglas technology:

$$\begin{aligned} Y_t &= Z_t F(K_t, EN_t) \\ &= Z_t (K_t)^{\alpha_k} (EN_t)^{1-\alpha_k}, \end{aligned}$$

where $K_t = \sum_{\tau=0}^{I-1} k_t^{t-\tau}$ represents aggregate capital and the inner product $EN_t = \sum_{\tau=0}^{I-1} h_t^{t-\tau} n_{1,t}^{t-\tau}$ represents total effective labor input. The parameter of the production function is assumed to satisfy $0 < \alpha_k < 1$. Also, it is assumed that total factor productivity, $z_t = \log(Z_t)$, evolves according to

$$z_t = \phi z_{t-1} + \sigma \epsilon_t, \tag{1}$$

and is to be calibrated in a subsequent section. The variable ϵ_t is white noise with unit variance and $\sigma > 0$ scales up the variance of the innovations.

Competitive pricing ensures all factors are paid their marginal products. The marginal productivity of an effective labor hour from the τth person will equal its real price, namely:

$$w_t = (1-\alpha_k) Z_t \left(K_t\right)^{\alpha_k} \left(EN_t\right)^{-\alpha_k}. \tag{2}$$

The marginal productivity of a unit of capital from the τth person will also equal its real price,

$$r_t = \alpha_k Z_t \left(K_t\right)^{\alpha_k - 1} \left(EN_t\right)^{1-\alpha_k}. \tag{3}$$

3.3 Characterization of the Equilibrium

The environment of the model is that a typical agent lives for I periods and makes a choice of consumption, savings, and leisure over her life. The formal dynamic programming problem of a household born at time t at time $t+\tau$ is

$$v_\tau(s_{t+\tau}^t, S_{t+\tau}) = \max\{u(c_{t+\tau}^t, n_{1,t+\tau}^t, n_{2,t+\tau}^t) + \beta \frac{\Psi_{\tau+1}}{\Psi_\tau} E_{t+\tau}[v_{\tau+1}(s_{t+\tau}^t, S_{t+\tau+1})]\},$$

where $s_{t+\tau}^t = \{k_{t+\tau}^t, h_{t+\tau}^t, a_{t+\tau}^t\}$ and $S_{t+\tau} = \{K_{t+\tau}, H_{t+\tau}, z_{t+\tau}\}$ subject to:

$$\begin{aligned} c_{t+\tau}^t + k_{t+\tau+1}^t &\leq (1 + r_{t+\tau} - \delta_k) k_{t+\tau}^t + w_{t+\tau} h_{t+\tau}^t n_{1,t+\tau}^t \\ h_{t+\tau+1}^t &\leq q_{t+\tau}^t + (1-\delta_h) h_{t+\tau}^t. \end{aligned}$$

The reader will notice that the aggregate state of the world contains only the mean values for K and H and not the entire distribution. While the next section of the paper will discuss this change in more detail, it suffices here for us to note that, using the results in Krusell and Smith (1998) and Young (2004), we are able to reduce the size of the state space considerably by ignoring higher-order moments. Indeed, it turns out that aggregate human capital ends up with a coefficient of zero in our computed policy functions, further reducing the computational burden of this model.

Given the behavior of the firm presented in the previous section, we are now in a position to define the equilibrium of the model.

Definition 1 (Recursive Competitive Equilibrium) *Given the stochastic process for Z and initial capital stocks, the competitive equilibrium is a set of pricing functions $\{w_t, r_t\}_{t=0}^{\infty}$; a set of human capital investments $\{\{q_t^{t-\tau}\}_{\tau=0}^{I-1}\}_{t=0}^{\infty}$; a set of consumptions $\{\{c_t^{t-\tau}\}_{\tau=0}^{I-1}\}_{t=0}^{\infty}$; capital allocations $\{\{k_t^{t-\tau},\, h_t^{t-\tau}\}_{\tau=0}^{I-1}\}_{t=0}^{\infty}$; a set of production plans for firms $\{K_t,\, EN_t\}_{t=0}^{\infty}$; a set of labor hour allocations $\{\{n_{1,t}^{t-\tau},\, n_{2,t}^{t-\tau}\}_{\tau=0}^{I-1}\}_{t=0}^{\infty}$; and a set of value functions $\{\{v_\tau(t)\}_{\tau=0}^{I-1}\}_{t=0}^{\infty}$ such that, given period 0 capital stocks, the following conditions are satisfied for all t:*

1. *The individual decisions are consistent with aggregate outcomes. That is, the supply of factors equals the firm's demand:*

$$K_t = \textstyle\sum_{\tau=1}^{I-1} k_t^{t-\tau}, \quad EN_t = \textstyle\sum_{\tau=0}^{I-1} h_t^{t-\tau} n_{1,t}^{t-\tau}.$$

2. *The allocations are feasible:*

$$C_t + K_{t+1} - (1-\delta_k)K_t = Z_t \left(K_t\right)^{\alpha_k} \left(EN_t\right)^{1-\alpha_k},$$

where $C_t = \sum_{\tau=0}^{I-1} c_t^{t-\tau}$, and

$$H_{t+1} - (1-\delta_k)H_t = \sum_{\tau=0}^{I-1} \exp(a_t^{t-\tau}) \left(h_t^{t-\tau}\right)^{\theta_1} \left(n_{2,t}^{t-\tau}\right)^{\theta_2},$$

where $H_t = \sum_{\tau=0}^{I-1} h_t^{t-\tau}$.

3. *Firms maximize pre-dividend stock market value period by period:*

$$\pi_t = \max\left\{Z_t \left(K_t\right)^{\alpha_k} \left(EN_t\right)^{1-\alpha_k} - r_t K_t - w_t EN_t\right\},$$

and factor prices are competitive. That is, the marginal productivity equations are satisfied.

4. Given the law of motion for the capital stocks, the price functions, initial conditions, and the transitions for the stochastic states, v is the solution of the following problem:

$$v_\tau(t+\tau) = \max\{u(c^t_{t+\tau}, n^t_{1,t+\tau}, n^t_{2,t+\tau}) + \beta\frac{\Psi_{\tau+1}}{\Psi_\tau}E_{t+\tau}[v_{\tau+1}(t+\tau+1)]\},$$

subject to: (i) the terminal condition $v_I(t) = 0$ for all t; (ii) nonnegativity conditions $c^t_t, k^t_{t+\tau}, h^t_{t+\tau} \geq 0$ for all t and τ; (iii) $k^t_t = 0$ and $k^t_{t+I} = 0$ for all t; and (iv) the budget constraints.

Optimal behavior of the household ensures the following set of Euler Equations and budget constraints hold at each time $t+\tau$ for each agent t:

$$\begin{aligned}
u_1(t+\tau) &= \beta\frac{\Psi_{\tau+1}}{\Psi_\tau}E_{t+\tau}\left\{u_1(t+\tau+1)(1+r_{t+\tau+1}-\delta_k)\right\} && \text{(4a)}\\
-u_2(t+\tau) &= u_1(t+\tau)h^t_{t+\tau}w_{t+\tau} && \text{(4b)}\\
-\frac{u_3(t+\tau)}{q^t_{n,t+\tau}} &= \beta\frac{\Psi_{\tau+1}}{\Psi_\tau}E_{t+\tau}\left\{\begin{array}{c} u_1(t+\tau+1)w_{t+\tau+1}n^t_{1,t+\tau+1}- \\ \frac{u_3(t+\tau+1)}{q^t_{n,t+\tau+1}}\left[q^t_{h,t+\tau+1}+(1-\delta_h)\right]\end{array}\right\}, && \text{(4c)}
\end{aligned}$$

where $u_1(t) = \partial u(\cdot)/\partial c_t$ and $u_2(t) = \partial u(\cdot)/\partial n_{1,t}$, for example. Equations (4a)-(4c) must hold at any time t for each consumer born at time $t-\tau$ where for $\tau \in \mathcal{I}$. These stochastic Euler equations, which are functions of the current and future states, are denoted by the following sets of equations:

$$\begin{aligned}
E_t\{SEE^\tau_a(s^{t-\tau}_t, s^{t-\tau}_{t+1}, S_t, S_{t+1}, \sigma)\}^{I-1}_{\tau=0} &= 0 && \text{(5a)}\\
E_t\{SEE^\tau_b(s^{t-\tau}_t, s^{t-\tau}_{t+1}, S_t, S_{t+1}, \sigma)\}^{I-1}_{\tau=0} &= 0 && \text{(5b)}\\
E_t\{SEE^\tau_c(s^{t-\tau}_t, s^{t-\tau}_{t+1}, S_t, S_{t+1}, \sigma)\}^{I-1}_{\tau=0} &= 0. && \text{(5c)}
\end{aligned}$$

It is important to note that (5a) and (5b) are standard but equations (5c) are nonstandard and represent the marginal costs of one more hour of skill acquisition equating with the expected discounted marginal benefit of an increase in skill hours. The marginal benefit is the increase in utility of skill acquisition discounted by the increase in next periods human capital. The discounted benefits are computed as the expected discounted increase in wage bill and the increase in human capital two periods forward. Note that the model has the feature that *learning begets learning*; skills acquired early facilitate later learning.

4 Calibration and Solution Method

4.1 Calibration

To conduct the quantitative analysis, the functional forms for utility must be set. Since there is no trend in hours worked in the data, but there is a trend in wages, the momentary utility function from the family of constant relative risk aversion is chosen:

$$u(c, n_1, n_2) = \frac{c^{1-\gamma}}{1-\gamma} + \varphi \frac{(1 - n_1 - n_2)^{1-\mu}}{1-\mu}.$$

The parameter γ is the Arrow-Pratt coefficient of relative risk aversion; $\gamma = 1.44$ is selected. The weight parameter on leisure is set to $\varphi = 0.75$, so the average fraction of time devoted to market activities is 0.33. The parameter μ determines the labor supply elasticity which is selected to match the calibration of Heathcote, Storesletten, and Violante (2004); their value is $\mu = 2.36$.

Secondly, the length of the life-cycle, I, must be set. In the OLG literature, it is common to assume that agents make economic decisions over, roughly, a 63 year period. If economic life is to start at 18 years of age, then the terminal age is 80 with retirement beginning at age 66. To keep computation of the equilibria tractable, however, the life-cycle is condensed so that $I = 21$. In this case, each period represents 3 years with retirement occurring at the end of period 16. Retirement represents the periods where labor hours have exogenously set to zero; $n_{1,t}^{17} = \ldots = n_{1,t}^{21} = 0$. As a result of the exogenously set retirement age, skill acquisition hours are $n_{2,t}^{16} = \ldots = n_{2,t}^{21} = 0$.

The calibrations for survival probabilites are presented in Table 2. These probabilities are found by converting the annual mortality probablities from the U.S. Life Tables of the National Center for Health Statistics (1992) to our $I = 21$ life-cycle.

The functional forms for the human capital transformation functions are calibrated from the empirical estimates in Heckman (1978) and Heckman et al. (1998). Heckman (1978) finds $\theta_1 = \theta_2 = 0.52$. For the shock to human capital production – also called a *schooling shock* – the following specification is considered:

$$\exp(a_t^{t-\tau}) = \bar{A}\exp(\alpha_h z_t),$$

where, by implication of (1), $E_{t-1}[z_t] = 0$. Heckman et al. (1998) find, when δ_h is set to zero, the human capital productivity technological variable $a_t^{t-\tau}$ mean at 0.081; this corresponds to $\bar{A} = 1.081^3 - 1 = 0.2632$ for a 21 period OLG model. The estimate for the initial human capital, $\bar{h}_t^t$, is found in Heckman et al. (1998); they estimate the mean initial human capital stock of an agent with a high-school education at $\bar{h}_t^t = 9.530$. The effects of z_t are normalized by setting $\alpha_h = 1.0$. In this case, an ability-to-pay for skills feature is included and is thus called the *ability-to-pay model*. The idea is that, presumably, resources for skill acquisition vary over the business cycle thus altering the productivity of hours and human capital.[6] In order to determine the quantitative effects of $\alpha_h = 1.0$, other calibrations of α_h are also considered; $\alpha_h = 0.0$ is the *benchmark model* and $\alpha_h = 1.50$ is the *high ability-to-pay model*.

The following set of parameters is left to be calibrated: $\{\alpha_k, \beta, \delta_k, \phi, \sigma, k_t^t\}$. First, $\alpha_k = 1/3$ is chosen which roughly matches the share of capital income in output for the United States since the Second World War. Next, the average discount rate β is fixed at a value

[6] In recessions, many institutions freeze hiring and reduce part-time faculty hours increasing class size thus making each skill hour less productive. Additionally, publically funded schools often raise tuition to meet funding shortfalls.

compatible with a yearly psychological rate of 3 percent; this gives $\beta = 1.03^{-3} = 0.9154$. The depreciation rate δ_k is set at 6 percent per year; thus the three year depreciation rate is

$$\delta_k = 1 - (1 - 0.06)^3 = 0.1694.$$

Finally, the parameters for the technology process z must be set. Most studies that calibrate the Solow residual series for an for an annual process set their standard deviations at 3 percent. The AR parameter is typically set so output's autocorrelation is 50 percent. This implies $\sigma = 0.01$ and $\phi = 0.50^3 = 0.125$. The results for all of the calibrations are in Table 3.

4.2 Perturbation

Since Kydland and Prescott (1982) and King, Plosser, and Rebelo (1988), it is common in macroeconomics to approximate the solution to nonlinear general equilibrium models using linear methods. However, the perturbation method of Judd (1998) and Gaspar and Judd (1997) has been recently shown by Schmitt-Grohé and Uribe (2004) to be a convenient higher-order solution method for the representative agent model. The novelty of perturbation applied to the OLG model is that the significant amounts of agent heterogeneity may cause dynamic properties of solutions to be nonlinear thus distorting the solutions. To date, the literature has focused on the transitions between steady states (Auerbach and Kotlikoff, 1987) and linear-quadratic economies (Ríos-Rull, 1999) due to the complexity of the model caused by the non-trivial heterogeneity, however.

The problem is obviously complex since the Euler Equations are functions of both the aggregate and private capital stocks. This can be handled by making the solutions, such as capital's *transition function* that relates today's state to tomorrow's, as a function of both the aggregate and the private capital stocks; let

$$k_{t+1}^{t-\tau} = \mathcal{K}^{\tau}(S_t^{\tau}), \tag{6}$$

where $S_t^{\tau} \equiv \{s_t^{t-\tau}, S_t, \sigma\}$. The remaining equilibrium solutions for market and non-market labor hours are denoted similarly:

$$\begin{aligned} n_{1,t}^{t-\tau} &= \mathcal{N}^{\tau}(S_t^{\tau}) &(7)\\ n_{2,t}^{t-\tau} &= \mathcal{Q}^{\tau}(S_t^{\tau}). &(8) \end{aligned}$$

Note that the *principle of optimality* has been invoked to show that an agent born at time $t+1$ should have the same demand for capital, all else equal, as an agent born at time t. This leads to the following aggregate capital definition: $K_{t+1} = \sum_{i=0}^{I-1} \mathcal{K}^i(S_t^i)$.[7]

In perturbation, a solution is an approximation to equations (6)-(8) by, for example, the second order Taylor series expansion:

$$\begin{aligned} \mathcal{K}^{\tau}(S_t^{\tau}) = {} & \mathcal{K}_0^{\tau} + \mathcal{K}_1^{\tau}(k_t^{t-\tau} - \bar{k}^{\tau}) + \mathcal{K}_2^{\tau}(h_t^{t-\tau} - \bar{h}^{\tau}) + \mathcal{K}_3^{\tau}(K_t - \bar{K}) + \\ & \mathcal{K}_4^{\tau}(H_t - \bar{H}) + \mathcal{K}_5^{\tau} z_t + \mathcal{K}_6^{\tau}\sigma + \\ & \frac{1}{2}\mathcal{K}_{1,1}^{\tau}(k_t^{t-\tau} - \bar{k}^{\tau})^2 + \frac{1}{2}\mathcal{K}_{2,2}^{\tau}(h_t^{t-\tau} - \bar{h}^{\tau})^2 + \ldots + \frac{1}{2}\mathcal{K}_{6,6}^{\tau}\sigma^2 + \\ & \mathcal{K}_{1,2}^{\tau}(k_t^{t-\tau} - \bar{k}^{\tau})(h_t^{t-\tau} - \bar{h}^{\tau}) + \ldots + \mathcal{K}_{5,6}^{\tau} z_t \sigma, \end{aligned}$$

where

$$\mathcal{K}_0^{\tau} = \mathcal{K}(S_t^{\tau})\big|_{S_t^{\tau} = \bar{S}^{\tau}}, \quad \mathcal{K}_1 = \left.\frac{\partial \mathcal{K}(S_t^{\tau})}{\partial k_t^{t-\tau}}\right|_{S_t^{\tau} = \bar{S}^{\tau}}, \quad \mathcal{K}_{1,1} = \left.\frac{\partial^2 \mathcal{K}(S_t^{\tau})}{\partial k_t^{t-\tau} \partial k_t^{t-\tau}}\right|_{S_t^{\tau} = \bar{S}^{\tau}}, \quad \ldots .$$

[7] This solution approach borrows its spirit from Krusell and Smith (1998). Young (2004) has applied their algorithm to overlapping generations economies with labor supply and factor taxation, finding that it performs quite well.

The higher order coefficients are found by repeated differentiation of equations (5a)-(5c) around the deterministic steady states of the economy. See Schmitt-Grohé and Uribe (2004) for further details.

Note that $\{\mathcal{K}_0^\tau\}_{\tau=0}^{I-1}$ are the steady state levels for the individual capital stocks. The steady state is found by solving the certainty version of equations (5a)-(5c) given by:

$$
\begin{aligned}
\{SEE_a^\tau(\bar{s}^\tau, \bar{s}^{\tau+1}, \bar{S}, \bar{S}, 0)\}_{\tau=0}^{I-1} &= 0 \\
\{SEE_b^\tau(\bar{s}^\tau, \bar{s}^{\tau+1}, \bar{S}, \bar{S}, 0)\}_{\tau=0}^{I-1} &= 0 \\
\{SEE_c^\tau(\bar{s}^\tau, \bar{s}^{\tau+1}, \bar{S}, \bar{S}, 0)\}_{\tau=0}^{I-1} &= 0.
\end{aligned}
$$

Finally, starting from $\bar{S}$ and given the solution coefficients $\{\mathcal{K}_0^\tau, \mathcal{N}_0^\tau, \ldots, \mathcal{Q}_{6,6}^\tau\}_{\tau=0}^{I-1}$, the time series for the allocations and prices are derived from 8000 simulated technology shocks; the first 1000 of each series are dropped to eliminate any transient dynamics.

5 Modeling Results

5.1 Steady State Profiles

Because our paper borrows heavily on parameter calibration from other studies, it is important to asses how well the life-cycle skill acquisition model replicates certain life-cycle facts from the literature. Specifically, average consumption is strongly humped-shaped with the peak around ages 50 (Fernández-Villaverde and Kruger, 2002; Gourinchas and Parker, 2002). Additionally, mean production hours are stable over the life-cycle, except a modest decline after age 50 (Heathcote et al., 2004). Finally, the rate of return to human capital investment declines with age (Carneiro and Heckman, 2003). Thus, skill acquisition hours should also be a decreasing function of age.

Figure 3 displays the certainty equivalence economy's ($\sigma = 0$) steady-state predictions for these profiles. The model can closely replicate two of the three steady state facts. First, consumption peaks at age 54-56; this is only two periods from the peak found in the data. Second, skill acquisition hours are a decreasing function of age. The model is only modestly able to replicate stability in market hours. Specifically, hours decrease by about 0.45 percent each year for 36 years till age 55. Then, market hours slightly increase towards retirement.

We conclude that, notwithstanding that our model doesn't include policy parameters such as social security or income taxation, the skill acquisition model gives reasonable life-cycle consumption, market production hours, and skill acquisition hours profiles that are often difficult to match.

5.2 Benchmark Model

Looking at the decision rule for hours used to accumulate skill when $\alpha_h = 0.0$, we see the coefficients on z are negative for all age groupings; the lowest and highest are $\mathcal{Q}_5^0 = -0.0120$ and $\mathcal{Q}_5^{14} = -0.0007$. Note that the coefficients have the interpretation of:

$$E[\partial \mathcal{Q}^\tau(S_t^\tau)/\partial z_t \,|\, S_t^\tau = \bar{S}^\tau] \equiv \mathcal{Q}_5^\tau;$$

these are the average partial effects from an increase in z on skill acquisition hours. Though the second-order effects are empirically important, in general, the effects of an increase in z can be estimated by examination of these partial effects. Therefore, because aggregate technology is the main determinate of cyclical fluctuations in output, the negative numbers imply that the economy's aggregate skill acquisition hours are countercyclical.

The first panel of Figure 4 plots the correlations between GDP and skill attainment hours for each age grouping for the simulated benchmark economy. Evidently, skill acquisition is countercyclical for all age groups; the substitution effect dominates the income effect from a positive aggregate technology shock. That is, when a boom in productivity occurs the intertemporally-high wage induces all households to shift hours to the now more productive goods sector despite the fact that human capital investment is a normal good.

Interestingly, the correlation between GDP and skill hours is an increasing function of age. This fact is straightforward to understand in the model. As households age, the willingness to substitute current leisure and consumption for future values diminishes, since there are now fewer periods in the future to recoup any utility losses. As a result, a household's willingness to supply additional units of time to skill acquisition becomes less dependent on the life-cycle and more on the business cycle – that is, it gets more closely tied to wages. Since the model predicts strongly procyclical wages, it also predicts increasing correlations between skill hours and output.

In total, these benchmark results are generally consistent with the aggregate data and the previous studies such as Perli and Sakellaris (2003). Additionally, the life-cycle model is generally consistent with the infinitely lived agent model studied by DeJong and Ingram (2001) where human capital acquisition is countercyclical. Though the model can explain aggregate behavior and retains the behavior found in the infinitely lived framework, the benchmark model is unable explain the procyclical behavior of the young's attainment of skills that is found in the data.

5.3 The Role of Ability-to-Pay

The fact that productivity of human capital attainment is assumed to be constant, $\bar{A} = A_t$ for all t, may not be a reasonable assumption. DeJong and Ingram (2001) found A_t to be positively correlated with aggregate total factor productivity Z_t. The effect of making A_t positively correlated with Z_t would most likely dampen the countercyclical response of skill acquisition hours. The idea is that the return (marginal cost) to human capital increases (decreases) during upturns thus making human capital a more attractive investment. Thus, the correlated productivity model is said to include an ability-to-pay feature.

The first panel of Figure 4 also plots the correlations between GDP and skill attainment hours for the remaining calibrations for α_h. The effect of increasing α_h is to increase the correlation of skill acquisition hours with respect to GDP. Figure 4 shows that the income effect increases for the younger age groups. Additionally, the income effect can become dominate for the young as shown by the high ability-to-pay model. It is also interesting to note that the correlations decline through the ages of 55. After age 55, the correlations between income and skill hours increase towards zero. Also in contrast to the benchmark, we see the coefficients on z can be both negative and positive. For example, the average of the first six coefficients for the calibration $\alpha_h = 1$ are positive; $\sum_{\tau=0}^{6-1} \mathcal{Q}_5^\tau/6 = 0.0042$. However, the effect of an increase in z on aggregate hours becomes increasingly smaller with $\sum_{\tau=0}^{I-1} \mathcal{Q}_5^\tau/I = 0.0028$.

These facts are also straightforward to understand in the model. Making A_t correlated with z_t increases the procycliality of the return to schooling. Therefore, in periods of contraction, the returns to human capital investment fall making those with the smallest ability-to-pay less likely to obtain schooling; this group are the wealth poor young. For the middle-aged, the ability-to-pay considerations are mitigated by their wealth and their own procyclical behavior when young; skill hours become increasingly countercyclical. Eventually, as households age, the willingness to substitute current leisure and consumption for future values diminishes, since there are now fewer periods in the future to recoup any utility losses (around age 55).

Previous empirical studies, such as Dellas and Sakellaris (2003), suggest that aggregate skill acquisition hours are countercyclical. Thus, it is natural to ask what are the cyclical properties of aggregate skill hours when the shocks to schooling are correlated with aggregate technology? In Table 4 we see that the benchmark model implies a strong negative correlation between aggregate skill acquisition hours and output at -0.9384. Whereas with $\alpha_h = \{1.0, 1.5\}$, the correlation is smaller at -0.4389 and -0.0946, respectively.

Interestingly, the cyclical properties of labor market hours are not significantly affected by the introduction of the ability-to-pay feature to skill acquisition; the correlation of market labor hours falls from 0.4813 to 0.4576 when $\alpha_h = 1.0$. For the high ability-to-pay model, the correlation of market labor hours falls similarly to 0.4450. As shown in the second panel of Figure 4, the slight reduction in the procyclical behavior of aggregate market labor hours is from a decrease in the correlation of the young's labor effort with aggregate income. Since market labor hours are strongly procyclical in the data, the retention of the behavior of market labor hours – that is a direct consequence of the elasticity supply of labor – is an important point worth noting.

5.4 The Skill Premium Over the Business Cycle

Recently, Young (2003) and Lindquist (2003) have found that the skill premium is essentially uncorrelated with income. Can ability-to-pay motives in skill acquisition explain this fact? Consider an example where skill acquisition is as in the first panel of Figure 4 benchmark model (*i.e.*, more countercyclical for the young). The young, who are relatively unskilled, would increase their labor effort in a boom at a higher rate than the old, who are relatively skilled. As a result, the skill premium, defined as the average wage rate of skilled to unskilled workers, would be countercyclical. Alternatively, suppose skill acquisition is as in the correlated models. During a productivity boom, the young leave the market labor force to obtain skills thus breaking the countercyclical behavior of the skill premium.

To compute a skill premium for the model, first, the 90th percent quantile steady state level of human capital is identified (roughly 10.21). Second, dummy variables are created to identify each period's skilled and unskilled labor based on the median steady state human capital level. Finally, a skill premium is calculated by taking the gross wage bill per labor hour of skilled to the gross wage bill per labor hour of the unskilled. Table 4 confirms that the countercyclical behavior of the skill premium is broken by the inclusion of the ability-to-pay feature in human capital accumulation; the correlation increases from -0.2011 in the benchmark to -0.1608 in the ability-to-pay model. In the high ability-to-pay model, we see that the skill premium has a correlation with income of -0.0294 indicating the premium is uncorrelated with the business cycle since the probability value is relatively high; $p = 0.0849$.

6 Conclusion

The model developed in this paper primarily focused on the role of human capital and the acquisition of it over the business cycle and life-cycle. Aggregate fluctuations affect skill acquisition hours through the existence of a human capital accumulation production sector of Heckman (1978) and Heckman et al. (1998). In the model, time is split between market production hours and skill acquisition hours.

The simulation results suggest that the substitution effect dominates the income effect for all age groups implying opportunity-cost considerations tend to make schooling countercyclical. However, the data on college enrollments suggests that ability-to-pay considerations, or the income effect, are more important for the very young since enrollments for the recently graduated from high-school are procyclical. The failure of the benchmark model is, probably, the result of making the human capital production function solely the function of each agent's private inputs. Including aggregates, such as total factor productivity, directly in the return to human capital accumulation can reverse the negative correlation result for the very young and thus replicate the business cycle behavior of the skill premium.

Most likely, returns to human capital accumulation are augmented with other aggregates that are indirectly influenced by total factor productivity. For example, tax revenues and, hence, subsidies for education are most likely procyclical. Thus, inclusion of total factor productivity in the returns to education may be replicating a correlation between TFP and the efficiency of skill acquisition production. The source of the correlation is a possible direction of future research.

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Figure 1: The first panel shows enrollment with a positive trend. The second panel shows variablity of enrollments attributed to 4 and 2 year institutions. The third panel indicates that undergraduate enrollments account for most of the variability in enrollments.

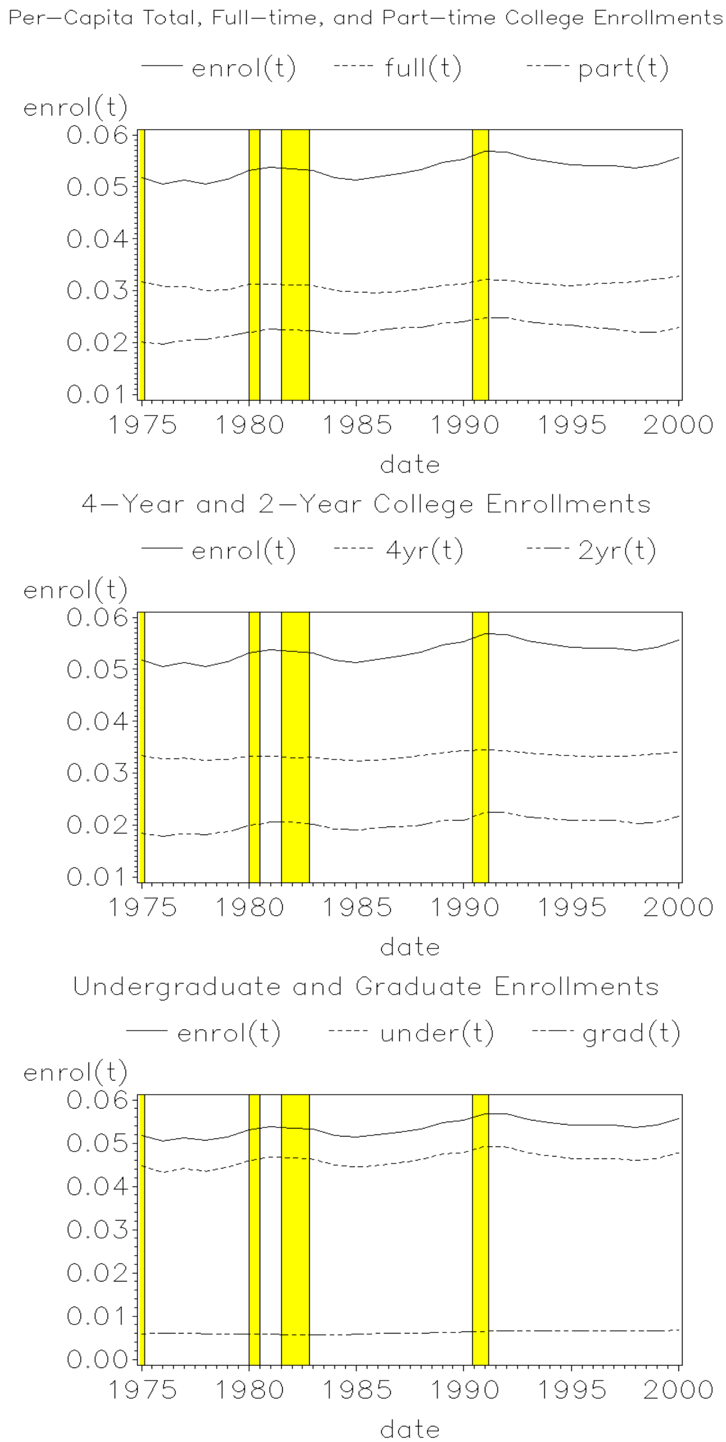

Figure 2: The cyclical component of college enrollment (total-recent high-school graduate) displays strong countercyclicity; $\rho = -0.57$. In the second panel, enrollment of recently graduated high-school seniors displays procyclicity; $\rho = 0.205$.

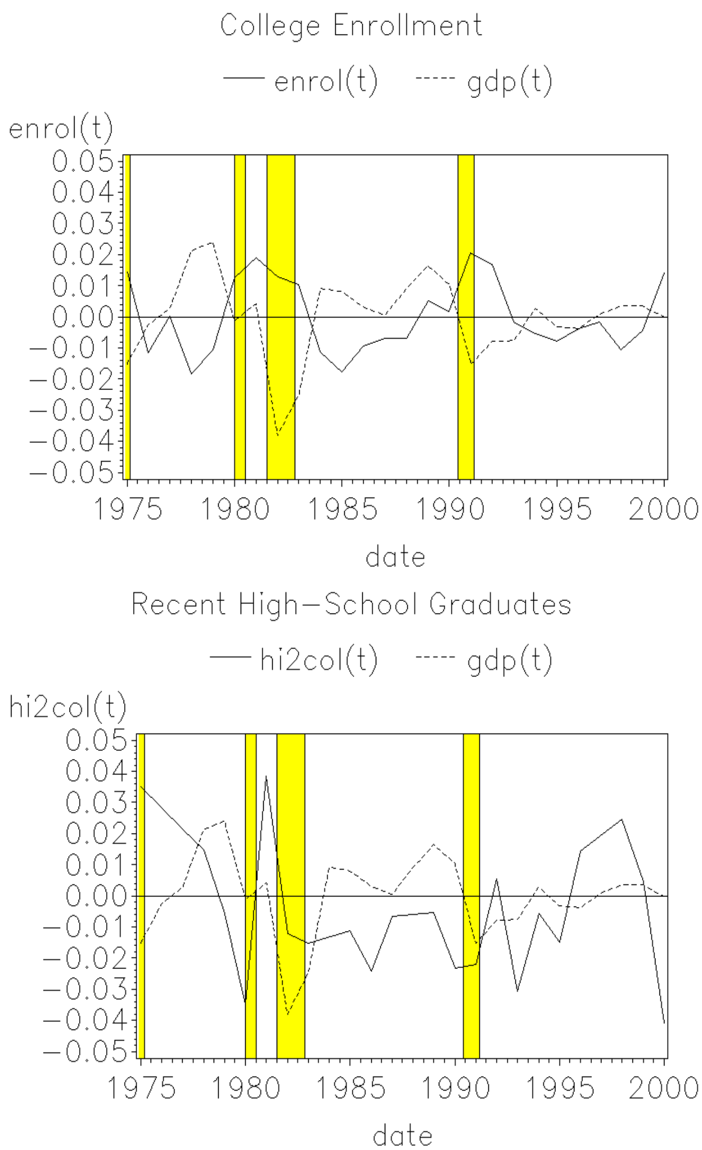

Figure 3: All profiles have been normalized to one by the peaks of each series.

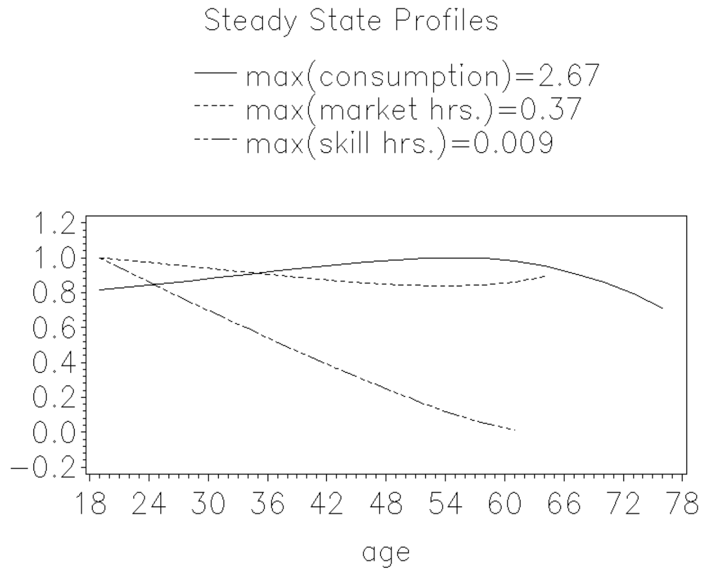

Figure 4: The first panel shows the correlation between the model's simulated GDP and skill aquisition hours for each age group. The Second panel shows the correlation between GDP and market hours.

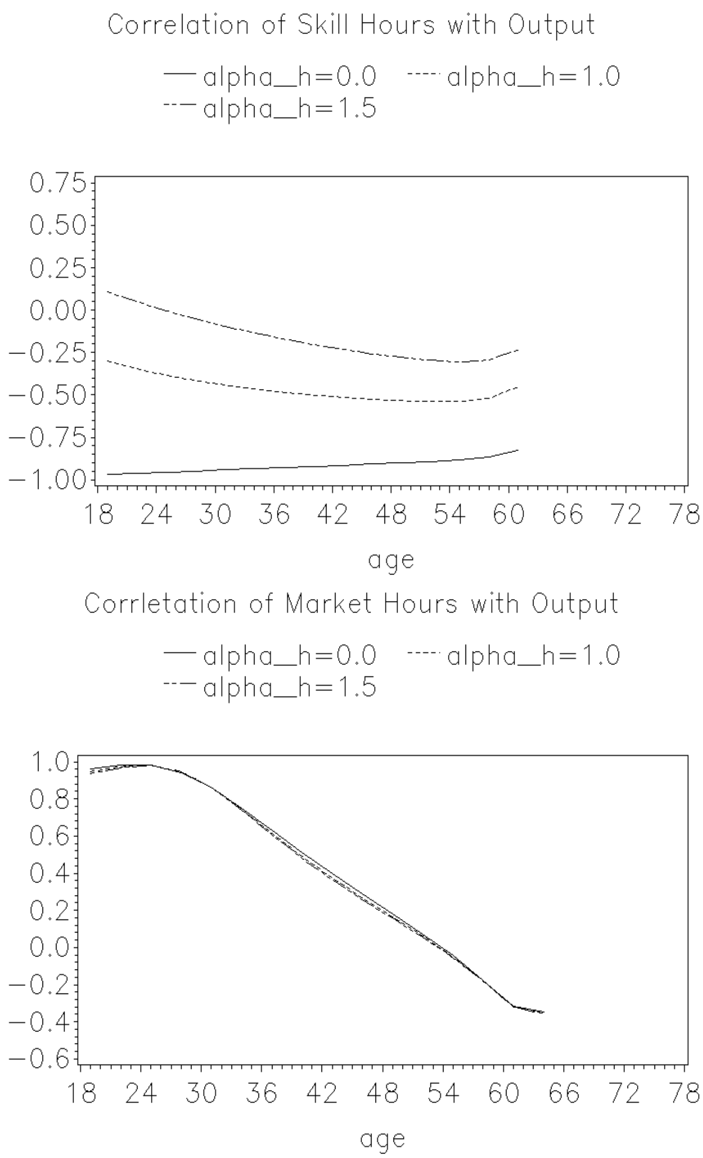

Table 1: Estimation results for the conditional linear feedback model. Stand-errors in parentheses and signifigance at 10 percent indicated by ($*$).

	ENROL	*HI2COL*
C	0.0566	-0.0992
	(.1142)	(.1130)
GDP	-0.6770*	0.9500*
	(.3743)	(.3619)
POP	-2.4684	8.4573
	(11.0929)	(10.6955)
γ	0.3654*	-0.1204
	(.1553	(.1853)
F	4.1380	1.7024

Note: $obj * N = 1.8891$

Table 2: Calibrations for survival probabilites.

$\psi_0 = 1.0000$	$\psi_1 = 0.9980$
$\psi_2 = 0.9975$	$\psi_3 = 0.9978$
$\psi_4 = 0.9982$	$\psi_5 = 0.9978$
$\psi_6 = 0.9968$	$\psi_7 = 0.9951$
$\psi_8 = 0.9926$	$\psi_9 = 0.9890$
$\psi_{10} = 0.9835$	$\psi_{11} = 0.9776$
$\psi_{12} = 0.9714$	$\psi_{13} = 0.9621$
$\psi_{14} = 0.9489$	$\psi_{15} = 0.9292$
$\psi_{16} = 0.9074$	$\psi_{17} = 0.8846$
$\psi_{18} = 0.8561$	$\psi_{19} = 0.8211$
$\psi_{20} = 0.7813$	$\psi_{21} = 0.7364$

Table 3: Calibration for benchmark and ability-to-pay model economies.

Preferences	$\beta = 0.9154, \gamma = 1.44, \mu = 2.36, \varphi = 0.75$
Technology	$\phi = 0.125, \sigma = 0.01$
Physical Capital	$\alpha_k = 0.333, \delta_k = 0.1694$
Human Capital	$\theta_1 = \theta_2 = 0.52, \bar{A} = 0.2632, \delta_h = 0.0$
Initial Stock	$k_t^t = 0, \bar{h}_t^t = 9.530$

Table 4: Simulated aggregate correlations with output (probability values in parantheses).

Variable	*Benchmark*	*Ability-to-Pay*	*High Ability-to-Pay*
Market Hours	0.4813 (p<.0001)	0.4576 (p<.0001)	0.4450 (p<.0001)
Skill Hours	−0.9388 (p<.0001)	−0.4389 (p<.0001)	−0.0946 (p<.0001)
Skill Premium	−0.2011 (p<.0001)	−0.1608 (p<.0001)	−0.0294 (p=.0849)