1. **Negate** the following statement: Some plinkers are slow.
   a. Not all plinkers are slow.
   b. Some plinkers are fast.
   c. Some plinkers are not slow.
   d. No plinkers are slow.
   e. None of the above

2. **Negate** the following statement: If the president lies, then the country prospers.
   a. If the country prospers, then the president lies.
   b. If the country does not prosper, then the president does not lie.
   c. If the president does not lie, then the country prospers.
   d. The president does not lie or the country prospers.
   e. The president lies and the country does not prosper.

3. Find the **contrapositive** of the following statement:
   No slow learners attend this school.
   a. If one does not attend this school, then one is a slow learner.
   b. If one attends this school, then one is not a slow learner.
   c. All slow learners attend this school.
   d. Some slow learners attend this school.
   e. A slow learner does attend this school.

4. Find the **contrapositive** of the following statement:
   If Michael drives fast, then Michael is happy.
   a. If Michael is happy, then Michael drives fast.
   b. If Michael drives fast, then Michael is not happy.
   c. If Michael is not happy, then Michael drives fast.
   d. If Michael is not happy, then Michael does not drive fast.
   e. If Michael does not drive fast, then Michael is happy.

5. Write the **converse** of the following statement: Every student at the university studies hard.
   a. The university studies every student hard.
   b. If one does not study hard, then one is not a student at the university.
   c. If one is a student at the university, then one does not study hard.
   d. If one studies hard, then one is a student at the university.
   e. None of the above is a converse of the statement.
6. **Negate** the following statement: If the road is slick, then driving is dangerous.

   a. If the road is not slick, then driving is not dangerous.
   b. If driving is dangerous then, then the road is slick.
   c. The road is slick and driving is not dangerous.
   d. Driving is dangerous when the road is not slick.
   e. Driving is dangerous and the road is not slick.

7. Find the **contrapositive** of the following statement:
   All good people are kind.

   a. If a person is not good, then the person is not kind.
   b. If a person is kind, the person is good.
   c. Some good people are kind.
   d. Some good people are unkind.
   e. All unkind people are not good.

8. Find the **converse** of the following statement:
   If Laura is happy, then Laura smiles.

   a. If Laura smiles, then Laura is happy.
   b. If Laura does not smile, then Laura is not happy.
   c. If Laura is happy, then Laura does not smile.
   d. Laura is not happy and Laura is not smiling.
   e. Laura is smiling or Laura is happy.

9. Write the following statement as an “if...,then...” statement
   Every student at the university studies hard.

10. Let $P$ denote the statement “The test is hard.” and let $Q$ denote the statement “The questions are easy.” Translate $Q \Rightarrow \neg P$ into a clearly written English sentence.
11. Let $P$ denote the statement “The test is hard.” and let $Q$ denote the statement “The student studies a lot.” Translate $\neg Q \lor \neg P$ into a clearly written English sentence.

12. Complete the following truth table.

<table>
<thead>
<tr>
<th>$P$</th>
<th>$Q$</th>
<th>$\neg P$</th>
<th>$\neg P \lor Q$</th>
<th>$P \lor (\neg P \lor Q)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>T</td>
<td>T</td>
<td>T</td>
<td>T</td>
<td></td>
</tr>
<tr>
<td>T</td>
<td>F</td>
<td>F</td>
<td>F</td>
<td></td>
</tr>
<tr>
<td>F</td>
<td>T</td>
<td>T</td>
<td>T</td>
<td></td>
</tr>
<tr>
<td>F</td>
<td>F</td>
<td>F</td>
<td>F</td>
<td></td>
</tr>
</tbody>
</table>

13. Complete the following truth table.

<table>
<thead>
<tr>
<th>$P$</th>
<th>$Q$</th>
<th>$\neg Q$</th>
<th>$P \land \neg Q$</th>
<th>$P \land \neg Q \Rightarrow Q$</th>
</tr>
</thead>
<tbody>
<tr>
<td>T</td>
<td>T</td>
<td>F</td>
<td>F</td>
<td></td>
</tr>
<tr>
<td>T</td>
<td>F</td>
<td>T</td>
<td>F</td>
<td></td>
</tr>
<tr>
<td>F</td>
<td>T</td>
<td>T</td>
<td>F</td>
<td></td>
</tr>
<tr>
<td>F</td>
<td>F</td>
<td>F</td>
<td>F</td>
<td></td>
</tr>
</tbody>
</table>

14. Complete the following truth table.

<table>
<thead>
<tr>
<th>$P$</th>
<th>$Q$</th>
<th>$P \land Q$</th>
<th>$P \land \neg Q$</th>
<th>$(P \land Q) \lor (P \land \neg Q)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>T</td>
<td>T</td>
<td>T</td>
<td>T</td>
<td></td>
</tr>
<tr>
<td>T</td>
<td>F</td>
<td>F</td>
<td>F</td>
<td></td>
</tr>
<tr>
<td>F</td>
<td>T</td>
<td>T</td>
<td>T</td>
<td></td>
</tr>
<tr>
<td>F</td>
<td>F</td>
<td>F</td>
<td>F</td>
<td></td>
</tr>
</tbody>
</table>

15. Complete the following truth table.

<table>
<thead>
<tr>
<th>$P$</th>
<th>$Q$</th>
<th>$\neg P$</th>
<th>$\neg P \lor Q$</th>
<th>$Q \land (\neg P \lor Q)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>T</td>
<td>T</td>
<td>T</td>
<td>T</td>
<td></td>
</tr>
<tr>
<td>T</td>
<td>F</td>
<td>F</td>
<td>F</td>
<td></td>
</tr>
<tr>
<td>F</td>
<td>T</td>
<td>F</td>
<td>F</td>
<td></td>
</tr>
<tr>
<td>F</td>
<td>F</td>
<td>F</td>
<td>F</td>
<td></td>
</tr>
</tbody>
</table>
16. Complete the following truth table.

<p>| | | | | | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>P</td>
<td>Q</td>
<td>R</td>
<td>P \Rightarrow Q</td>
<td>Q \Rightarrow R</td>
<td>P \Rightarrow Q \land Q \Rightarrow R</td>
</tr>
</tbody>
</table>

17. Given the premises $P \Rightarrow Q$, $\neg P \Rightarrow \neg S$, $R \Rightarrow \neg Q$, prove $R \Rightarrow \neg S$.

1. $R$ (hypothesis)
2.
3.
4.
5.
6.
7.

18. Given the premises $P$, $P \Rightarrow \neg Q$, $Q \lor R$, prove $R$.

19. Given $(P \lor \neg P) \Rightarrow Q$, $\neg Q \lor R$ prove $R$. 
20. Prove or disprove: \( P \lor (\neg P \lor Q) \) is a tautology.

21. Use induction to prove \( D^n(x^n) = n! \) for \( n = 1, 2, 3, \ldots \).

22. Use induction to prove the identity

\[
\sum_{j=1}^{n} j^3 = \frac{1}{4} n^2(n + 1)^2 \text{ for } n = 1, 2, 3, \ldots.
\]
23. Use induction (on $n$) to prove \[ \sum_{j=0}^{n} \left( \frac{1}{2} \right)^j = 2 - \left( \frac{1}{2} \right)^n \] for $n = 0, 1, 2, \ldots$.

24. Negate the following statement: $(\forall x)(\forall y)(x + y > x)$.

25. Find the solution set for the following predicate when $U = \mathbb{R}$.

\[ |x - 3| \leq 5 \]

26. (True or False). A tautology can be made False by changing the truth value of some of its constituent prime statements.

True    False
27. (True or False). Let $M$ denote the set of all living men, and let $W$ denote the set of all women who have ever existed. Determine whether the following statement is true or false.

$$ \exists w \in W, \forall m \in M, m \text{ is a child of } w $$

True    False

28. (True or False) Let $O$ denote a statement that is always false. From the premise $p$ and the premise $(p \land \neg q) \Rightarrow O$, we can prove $q$.

True    False

29. (True or False) Let $U = \mathbb{R}$. If $x < y$ and $xy \neq 0$, then $1/y > 1/x$.

True    False

30. (True or False) Let $U = \mathbb{R}$. $\exists m \forall x \left( \frac{x}{|x|+1} > m \right)$

True    False