1. Let the universal set $U$ be given by $U = \{1, 2, 3, 4, 5, 6, 7, 8, 9, 10\}$.
   If $A = \{1, 2, 3, 4, 5, 6\}$ and $B = \{1, 3, 5, 7, 9\}$, find $A \cap B'$.
   $$A \cap B' = \{2, 4, 6\}$$

2. Let the universal set $U$ be given by $U = \{1, 2, 3, 4, 5, 6, 7, 8, 9, 10\}$.
   If $A = \{1, 2, 3, 4, 5, 6\}$ and $B = \{1, 3, 5, 7, 9\}$, find $A' \cup B$.
   $$A' \cup B = \{1, 3, 5, 7, 8, 9, 10\}$$

3. Let the universal set $U$ be given by $U = \{1, 2, 3, 4, 5, 6, 7, 8, 9, 10\}$.
   If $A = \{1, 2, 3, 4, 5, 6\}$, $B = \{1, 3, 5, 7, 9\}$, and $C = \{4, 7, 10\}$, find $(A - C) \cup B$.
   $$(A - C) \cup B = \{1, 2, 3, 5, 6, 7, 9\}$$

4. Given the premises $P \lor Q \Rightarrow S$ and $R \Rightarrow P \land Q$, one can prove $R \Rightarrow S$.
   Find an analogous theorem involving sets $A$, $B$, $C$, and $D$.
   
   (a) If $A \subseteq B$, $B \subseteq C$, and $C \subseteq D$, then $A \subseteq D$.
   (b) If $A \subseteq B$, $A' \cap C \subseteq D$, and $B' = U$, then $C \subseteq D$.
   (c) If $A \cup B \subseteq D$ and $C \subseteq A \cap B$, then $C \subseteq D$.
   (d) If $A \subseteq B$, $A' \cap C \subseteq D$, and $B' = \emptyset$, then $C \subseteq D$.
   (e) If $A \subseteq B$, $A' \cap C \subseteq D$, and $C \subseteq B$, then $C \subseteq D$.

5. Below is a Venn diagram. How would you denote the set represented by the shadowed regions?

   Answer: $C \cap (A \cup B) - (A \cap B)$.
   also $CAB' \cup CA' B$
6. Express \((A \cap B') \cup (A \cap B)\) in its simplest form.

\[
(A \cap B') \cup (A \cap B) = A \cap (B' \cup B) = A \cap \emptyset = A
\]

Answer: \(A\)

7. Find the theorem which has the following proof.
Proof. Assume \(A \subseteq B\) and \(B \subseteq C\). Let \(x \in C'\). Then, by definition of set complement, \(x \notin C\). Since \(B \subseteq C\), \(x \notin C\) implies \(x \notin B\) and, since \(A \subseteq B\), \(x \notin B\) implies \(x \notin A\). Thus \(x \in A'\). \(\square\)

Theorem. If \(A \subseteq B\) and \(B \subseteq C\), then \(C' \subseteq A'\).

8. Let \(\Omega = \{I_n = [2 - \frac{1}{n}, 3 + \frac{1}{n}] : n \in \mathbb{Z}^+\}\) where \(\mathbb{Z}^+\) denotes the set of positive integers. Find \(I_1\), \(I_2\), and \(I_3\).

\[
I_1 = [1, 4]
\]
\[
I_2 = [\frac{3}{2}, \frac{7}{2}]
\]
\[
I_3 = [\frac{5}{3}, \frac{10}{3}]
\]

9. Let \(\Omega = \{I_n = [2 - \frac{1}{n}, 3 + \frac{1}{n}] : n \in \mathbb{Z}^+\}\) where \(\mathbb{Z}^+\) denotes the set of positive integers. Find \(\cap \Omega\).

Note \(\lim_{n \to \infty} (2 - \frac{1}{n}) = 2\) and \(\lim_{n \to \infty} (3 + \frac{1}{n}) = 3\)

a. \(\emptyset\)
b. \([2, 3]\)
c. \([1, 4]\)
d. \([2, 3]\)
e. none of the above

10. Let \(A = \{0, 1, 2\}\) and \(B = \{-1, 0, 1\}\). How many ordered pairs does \(A \times A\) have?

a. 0
b. 3
c. 6
d. 9
e. 27
11. Let $A = \{a, b, c\}$. Find the power set of $A$.

$$\mathcal{P}(A) = \{\emptyset, \{a\}, \{b\}, \{c\}, \{a,b\}, \{a,c\}, \{b,c\}, A\}$$

12. Recall that $\mathcal{P}(S)$ denotes the power set of $S$.
If $A = \{a, b, c\}$, find the size (cardinality) of the set $\mathcal{P}(\mathcal{P}(A))$.

$$2^3 = 2^8 = 256$$

Answer: $256$

13. Complete the proof below for the theorem which states $A \cap (A \cup B) = A$.
Use the facts 1. $A = (A \cap B') \cup (A \cap B)$ and 2. $C \cup C = C$

Proof. $A \cap (A \cup B) = (A \cap A) \cup (A \cap B)$ (Distributive law)

$= A \cup (A \cap B)$ (since $A \cap A = A$)

$= ((A \cap B') \cup (A \cap B)) \cup (A \cap B)$ (by fact 1)

$= (A \cap B') \cup ((A \cap B) \cup (A \cap B))$ (Associativity)

$= (A \cap B') \cup (A \cap B)$ (by fact 2)

$= A$ (by fact 1) \[\square\]

14. Let $A$, $B$, and $C$ be subsets of some universal set $\mathcal{U}$.
Complete the Venn diagram for $(A \cap B) \cap C'$. 

![Venn Diagram](image)
15. Complete the following table of analogous theorems.

<table>
<thead>
<tr>
<th>Logic</th>
<th>Set Theory</th>
</tr>
</thead>
<tbody>
<tr>
<td>( P \land \neg P \equiv \neg \neg P \</td>
<td>A \cap A' = \emptyset</td>
</tr>
<tr>
<td>( P \Rightarrow Q \equiv \neg P \lor Q \</td>
<td>A \subseteq B \iff A' \cup B = \mathcal{U}</td>
</tr>
<tr>
<td>( P \lor Q \Rightarrow R \iff (P \Rightarrow R) \land (Q \Rightarrow R) \</td>
<td>A \cup B \subseteq C \iff A \subseteq C \text{ and } B \subseteq C</td>
</tr>
<tr>
<td>( P \Rightarrow Q \iff \neg Q \Rightarrow \neg P \</td>
<td>A \subseteq B \iff B' \subseteq A'</td>
</tr>
<tr>
<td>( (P \Rightarrow Q) \lor P \equiv \top \</td>
<td>(A' \cup B) \cup A = \mathcal{U}</td>
</tr>
<tr>
<td>( (P \Rightarrow Q) \land (Q \Rightarrow S) \Rightarrow (P \Rightarrow S) \</td>
<td>\text{If } A \subseteq B \text{ and } B \subseteq C, \text{ then } A \subseteq C</td>
</tr>
</tbody>
</table>

16. Let \( f = \{(0, 3), (1, 1), (2, 2), (3, 0)\} \). Find \( f^{-1} \).

\[
\begin{align*}
f^{-1} = \{(3, 0), (1, 1), (2, 2), (0, 3)\}
\end{align*}
\]

17. Let \( f = \{(0, 3), (1, 1), (2, 2), (3, 0)\} \). Find \( (f \circ f) \).

\[
(f \circ f) = \{(0, 0), (1, 1), (2, 2), (3, 3)\}
\]

18. Let \( \mathcal{N} = \{0, 1, 2, \ldots\} \). Let \( f : \mathcal{N} \to \mathcal{N} \) be defined by \( f(n) = 2n + 1 \).
Since \( f \) is a one-to-one function, \( f \) has an inverse. Find \( f^{-1}(5) \).

Note \( f(2) = 5 \).

\[
\begin{align*}
f^{-1}(5) = 2.
\end{align*}
\]

19. Let \( f : \mathcal{R} \to \mathcal{R} \) be defined by \( f(x) = \frac{1}{x+1} \). Find the range of \( f \). \( \text{Range}(f) = (0, 1] \)

a. range of \( f = \{y \in \mathcal{R} : y > 0\} \)
b. range of \( f = \{y \in \mathcal{R} : y \geq 0\} \)
c. range of \( f = \{y \in \mathcal{R} : y \neq 0\} \)
d. range of \( f = \{y \in \mathcal{R} : y < 0\} \)
e. none of the above

20. Let \( f : \mathcal{R} \to \mathcal{R} \) be defined by \( f(x) = x^3 - 3x + 1 \). If \( A = [-1, 1] \), find \( f[A] \).

a. \([-1, -1]\]  
   \text{Note } f \text{ is decreasing on } A \text{ with } f(-1) = 3 \text{ and } f(1) = -1.
b. \([-3, 1]\]  
c. \([0, -1]\]  
d. \([-1, 3]\]  
e. none of the above
21. Complete the proof below that the function \( f : \mathcal{R} \to \mathcal{R} \) defined by \( f(x) = 3x - 1 \) is injective.
Proof. Let \( f(a) = f(b) \). Then \( 3a - 1 = 3b - 1 \). Hence \( a = b \). \( \square \)

22. Complete the proof below that \( (A \cap B) - (A \cup B) = \emptyset \).
Proof. \( (A \cap B) - (A \cup B) = (A \cap B) \cap (A \cup B)' \) [by set difference definition]
\( = (A \cap B) \cap (A' \cap B') \) [by de Morgan law]
\( = (A \cap A') \cap (B \cap B') \) [by commutativity and associativity]
\( = \emptyset \cap \emptyset \) [by complement law]
\( = \emptyset \) [by identity law] \( \square \)

23. Prove \( A - (A - B) = A \cap B \).
Proof. \( A - (A - B) = A - (A \cap B') \) [by set difference definition]
\( = A \cap (A \cap B')' \) ["]
\( = A \cap (A' \cup B) \) [by de Morgan law]
\( = (A' \cap A) \cup (A \cap B) \) [by distributivity]
\( = \emptyset \cup (A \cap B) \) [by complement law]
\( = A \cap B \) [by identity law] \( \square \)

24. Prove \( (A - B) \cup B = A \cup B \).
Proof. \( (A - B) \cup B = (A \cap B') \cup B \) [set difference]
\( = (A \cup B) \cap (B' \cup B) \) [distributivity]
\( = (A \cup B) \cap \emptyset \) [complement law]
\( = A \cup B \) [identity law] \( \square \)

25. Complete the proof below that \( A \subseteq f^{-1}[f[A]] \).
Proof. Let \( x \in A \). By definition of \( f[A] \), \( x \in A \) implies \( f(x) \in f[A] \).
By the definition of the pre-image of a set under \( f \), \( f(x) \in f[A] \)
implies \( x \in f^{-1}[f[A]] \), and we are done. \( \square \)