1. Let $X_1$ and $X_2$ be independent binomial random variables where $X_1 \sim \text{Binomial}(n = 10, p = .2)$ and $X_2 \sim \text{Binomial}(n = 5, p = .2)$. Find the conditional probability $P(X_1 = 2 \mid X_1 + X_2 = 3)$.

2. Suppose a door-to-door salesperson has probability .10 of successfully completing a sale at each house visited. What is the expected number of houses the salesperson must visit in order to complete 3 successful sales? (Assume the outcomes among all the houses visited are independent.)

3. Let $X$ denote the number of automobile accidents per week at intersection A and let $Y$ denote the number of automobile accidents per week at intersection B. If $X$ and $Y$ are independent Poisson random variables with respective means 3 and 5, calculate the conditional expected value of $X$ given that $X + Y = 10$.

4. The joint density of $X$ and $Y$ is given by $f(x, y) = \frac{1}{2}ye^{-xy} I_{(0,\infty)}(x) I_{(0,2)}(y)$. Find $E[e^{X/3} \mid Y = \frac{2}{3}]$.

5. An unbiased die is successively rolled. Let $X$ denote the number of rolls necessary to obtain a five, and let $Y$ denote the number of rolls necessary to obtain a six. Find (a) $E[X]$, (b) $E[X \mid Y = 1]$, (c) $E[X \mid Y = 3]$.

6. Let $X$ have probability density function given by $f(x) = \frac{1}{2}e^{-x/2} I_{(0,\infty)}(x)$. Find $E[X \mid X > 2]$.

7. Two players (A and B) alternate rolling a fair die. The first one to obtain a six is declared the winner. Find the probability that the first player to roll (Player A) is the winner.

8. Suppose that you successively roll a fair die until the sum of all throws exceeds $k$. Let $N_k$ denote the number of rolls needed. Find $E[N_k]$ for $k = 1, 2,$ and $3$. 


9. Suppose \( N \) is a Poisson random variable with mean \( \lambda = 2 \). Let \( X_1, X_2, \ldots \) be independent and identically distributed Binomial(10, .5) random variables. For 
\[
S = \sum_{i=1}^{N} X_i,
\]
find \( E(S) \) and \( \text{Var}(S) \).

10. Four red and two blue balls are distributed in the urns in such a way that each contains three balls. We say that the system is in state \( i \), \( i = 1, 2, 3 \) if the first urn contains \( i \) red balls. At each step, we draw one ball from each urn and place the ball drawn from the first urn into the second, and conversely with the ball from the second urn. Let \( X_n \) denote the state of the system after the \( n \)th step.

(i) Explain why \( \{X_n, n = 0, 1, 2, \ldots\} \) is a Markov chain.

(ii) Calculate the transition probability matrix \( \mathbb{P} \).

(iii) Find \( P_{13}^5 \).

(iv) Prove that state 3 is a recurrent state.

(v) Find the long-run proportion of time that the system is in state 2.

11. Consider a Markov chain whose state space is \( \mathbb{Z} \), the set of integers, and has transition probabilities given by \( P_{i,i+1} = .75 \) and \( P_{i,i-1} = .25 \).

(i) Find \( P_{00}^2 \), \( P_{00}^4 \), and \( P_{00}^6 \).

(ii) Find \( P_{00}^{2n} \).

(iii) Prove that state 0 is a transient state.

(iv) Find the expected number of time periods the process will be in state 0 given that it started in state 0.

(v) Find probability that, starting in state 0, the process will never again enter state 0.