

## Cox and Stuart Test for Trend

A series of observations is said to exhibit an upward trend if the magnitudes of the later observations tend to be greater than those of the earlier observations. The data exhibit a downward trend if the earlier observations tend to be larger than the later observations. In 1955, Cox and Stuart introduced a test for trend that is based on the sign test (See Cox, D.R. and Stuart, A. Some quick tests for trend and dispersion. *Biometrika*, 42, 80-95.)

**Example** (from Daniel text): The Federal Crop Insurance Corporation's *Annual Report to Congress* for 1973 contains the information on cotton crop insurance shown in the table below. Do these data indicate a downward trend in the number of cotton crops insured? Use level of significance  $\alpha = .10$ .

Number of U.S. Crops insured each year, 1948 – 1972

| Year | Crops Insured | Year | Crops Insured | Sign of $X_{i+12} - X_i$ |
|------|---------------|------|---------------|--------------------------|
| 1948 | 19,479        | 1961 | 25,375        | +                        |
| 1949 | 26,667        | 1962 | 21,312        | -                        |
| 1950 | 63,969        | 1963 | 26,526        | -                        |
| 1951 | 57,715        | 1964 | 24,865        | -                        |
| 1952 | 38,086        | 1965 | 21,152        | -                        |
| 1953 | 38,434        | 1966 | 23,458        | -                        |
| 1954 | 24,196        | 1967 | 25,774        | +                        |
| 1955 | 19,319        | 1968 | 32,646        | +                        |
| 1956 | 29,975        | 1969 | 31,786        | +                        |
| 1957 | 25,451        | 1970 | 24,821        | -                        |
| 1958 | 20,410        | 1971 | 19,593        | -                        |
| 1959 | 19,910        | 1972 | 14,960        | -                        |
| 1960 | 15,628        |      |               |                          |

Pair the  $i$ th observation of the first half with the  $i$ th observation of the second half of the time-ordered data. Since there are 25 observations above, the middle value does not get paired. Assign "+" or "-" to each of the pairs  $(X_i, X_{i+12})$ . If  $X_{i+12} > X_i$ , assign a "+". If  $X_{i+12} < X_i$ , assign a "-". Let  $T$  = number of "+" pairs. Under the null hypothesis of no trend,  $T \sim \text{Binomial}(12, 1/2)$ . Now if  $T$  is observed to take on a sufficiently small value, we have significant evidence of a downward trend. Here  $T$  is 4. The p-value is  $P(T \leq 4 | H_0 \text{ is true}) = .1938$ . Since the p-value exceeds  $\alpha = .10$ , we do not reject  $H_0$ .

**Example.** Mean mileage of cars 1970 – 1983. Data from U.S. Dept. of Commerce. Test for a trend. Use  $\alpha = .05$ .

| Year | Mileage (thousands) | Year | Mileage |
|------|---------------------|------|---------|
| 1970 | 9.8                 | 1977 | 9.6     |
| 1971 | 9.9                 | 1978 | 9.8     |
| 1972 | 10.0                | 1979 | 9.3     |
| 1973 | 9.8                 | 1980 | 8.9     |
| 1974 | 9.2                 | 1981 | 8.7     |
| 1975 | 9.4                 | 1982 | 9.2     |
| 1976 | 9.5                 | 1983 | 9.3     |

## Using the trend test for a correlation test.

**Example.** Twelve randomly selected students from a large statistics class were asked on the first day of class to estimate the average number of hours they studied each day. The first exam scores of the 12 students along with their study hours are given below. Is there evidence of a significant positive correlation between exam score and number of study hours ?

|              |     |    |     |     |    |     |     |    |    |     |    |     |
|--------------|-----|----|-----|-----|----|-----|-----|----|----|-----|----|-----|
| Student:     | 1   | 2  | 3   | 4   | 5  | 6   | 7   | 8  | 9  | 10  | 11 | 12  |
| Study hours: | 3.5 | 2  | 4.3 | 1.5 | 4  | 2.5 | 1.5 | 3  | 1  | 4.5 | 5  | 3.3 |
| Exam score:  | 94  | 73 | 90  | 70  | 88 | 75  | 60  | 81 | 69 | 97  | 91 | 75  |

**Example.** (from Iman&Conover, p336). Test for positive correlation. Use  $\alpha = .10$ .

| <u>Company</u>  | <u>Institutional Shares (thous.)</u> | <u>Earnings Per Share</u> |
|-----------------|--------------------------------------|---------------------------|
| Danners         | 197                                  | 1.28                      |
| Duckwall-Alco   | 24                                   | 2.20                      |
| Fed-Mart        | 99                                   | 4.80                      |
| Grand Central   | 2                                    | 1.22                      |
| Hartfield-Zodys | 377                                  | 1.97                      |
| K-Mart          | 53,170                               | 2.96                      |
| Thrifty Corp    | 544                                  | 1.46                      |
| Wal-Mart        | 3,452                                | 2.34                      |
| Woolworth       | 6,587                                | 4.84                      |
| Zayre           | 632                                  | 2.64                      |