When checking the normality assumption for error terms in a regression model, one often makes use of a normal probability plot of the residuals. A normal probability plot is a plot of ordered sample data versus their estimated expected (standardized) values if the sample came from a normal population. A strong linear trend in the plot is indicative of normality. The linearity of the plot can be measured by the correlation coefficient \( r \) for the ordered residuals and their normal scores, i.e., \( r = \text{corr} \left( e(i), z \left( \frac{i}{n+0.25} \right) \right) \). The quantity \( z_j \) represents the \( j \) quantile from a standard normal distribution.

**Example 1.** Suppose we have the following residuals (after ordering): -11, -9, -2, 5, 8, 9

<table>
<thead>
<tr>
<th>( i )</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \left( \frac{i}{n+0.25} \right) )</td>
<td>.10</td>
<td>.26</td>
<td>.42</td>
<td>.58</td>
<td>.74</td>
<td>.90</td>
</tr>
<tr>
<td>normal scores: ( z \left( \frac{i}{n+0.25} \right) )</td>
<td>-1.28155</td>
<td>-.64335</td>
<td>-.20189</td>
<td>.20189</td>
<td>.64335</td>
<td>1.28155</td>
</tr>
<tr>
<td>residuals: ( e(i) ) (standard)</td>
<td>-11, (-1.268)</td>
<td>-9, (-1.038)</td>
<td>-2, (-.231)</td>
<td>5, (.577)</td>
<td>8, (.923)</td>
<td>9 (1.038)</td>
</tr>
</tbody>
</table>

**Normal Probability Plot of Standardized Residuals**

Linearity of the normal probability plot supports the assumption of normal error terms. Nonlinearity provides evidence of non-normal error terms. The plot above indicates some nonlinearity, but it might not be statistically significant due to the small sample size.
A test for normality is provided in Kutner et al., based on critical values from a paper by Looney and Gulledge, “Use of the correlation coefficient with normal probability plots,” The American Statistician 39 (1985), pp.75-79.

Test

The hypotheses are given by

\[ H_0: \text{The error terms are normally distributed} \]
\[ H_A: \text{The error terms are non-normal}. \]

Decision rule: The null hypothesis is rejected at the \( \alpha \) level of significance if the test statistic \( r \) is less than the critical value \( r_\alpha \) provided by Looney and Gulledge (and presented in the table below).

Critical values for Correlation Coefficient between Ordered Residuals and “Expected” Values under \( H_0 \)

<table>
<thead>
<tr>
<th>n</th>
<th>.10</th>
<th>.05</th>
<th>.01</th>
<th>(approximation for ( \alpha = .05 )) ***</th>
</tr>
</thead>
<tbody>
<tr>
<td>5</td>
<td>.903</td>
<td>.880</td>
<td>.826</td>
<td>.879</td>
</tr>
<tr>
<td>10</td>
<td>.934</td>
<td>.918</td>
<td>.879</td>
<td>.920</td>
</tr>
<tr>
<td>15</td>
<td>.951</td>
<td>.939</td>
<td>.910</td>
<td>.938</td>
</tr>
<tr>
<td>20</td>
<td>.960</td>
<td>.951</td>
<td>.926</td>
<td>.949</td>
</tr>
<tr>
<td>25</td>
<td>.966</td>
<td>.959</td>
<td>.939</td>
<td>.957</td>
</tr>
<tr>
<td>30</td>
<td>.971</td>
<td>.964</td>
<td>.947</td>
<td>.962</td>
</tr>
<tr>
<td>40</td>
<td>.977</td>
<td>.972</td>
<td>.959</td>
<td>.970</td>
</tr>
<tr>
<td>50</td>
<td>.981</td>
<td>.977</td>
<td>.966</td>
<td>.975</td>
</tr>
<tr>
<td>75</td>
<td>.987</td>
<td>.984</td>
<td>.976</td>
<td>.983</td>
</tr>
<tr>
<td>100</td>
<td>.989</td>
<td>.987</td>
<td>.982</td>
<td>.988</td>
</tr>
</tbody>
</table>

*** Note that the values in the second column (\( \alpha = .05 \)) can be closely approximated by the value in the last column obtained by the following formula derived by Dennis Walsh:

\[ r_{.05, n} \approx 1.02 - \frac{1}{\sqrt{10n}}. \]

Note that \( r_{.05, 25} = .959 \) from table, while \( r_{.05, 25} \approx 1.02 - \frac{1}{\sqrt{250}} \approx .957 \) from formula.

**Example 1 (continued).** The correlation coefficient for the (standardized) residuals and their normal scores in example 1 is calculated to be \( r = .956 \) Using \( \alpha = .10 \), the critical value (interpolated) from the table above is \( r_{.05, 6} = .909 \). Since .956 > .909, we do not have sufficient evidence to conclude that the error terms have a non-normal distribution. The error terms could have a normal distribution.