A binary matrix has entries of either 0 or 1. A symmetric matrix $A$ is a square matrix such that $A = A^T$, that is, $A$ is symmetric about its main diagonal. A bisymmetric matrix $B$ is symmetric about both main diagonals. For example, the following binary matrix is bisymmetric:

$$
\begin{bmatrix}
  1 & 0 & 1 & 0 \\
  0 & 0 & 0 & 1 \\
  1 & 0 & 0 & 0 \\
  0 & 1 & 0 & 1 \\
\end{bmatrix}
$$

A square $n \times n$ matrix $A = [a_{ij}]$ is bisymmetric if $a_{ij} = a_{ji}$ and $a_{ij} = a_{n-j+1,n-i+1}$. For example in the matrix above we have $a_{13} = a_{31} = a_{24} = 1$.

We will derive the number of $n \times n$ binary bisymmetric matrices. Let $a(n)$ denote the count. First we define the null matrix to be a $0 \times 0$ matrix with no rows and no columns. The null matrix will vacuously not fail to satisfy the properties of a binary bisymmetric matrix and hence $a(0) = 1$. We note that $a(1) = 2$ since both [0] and [1] are $1 \times 1$ bisymmetric matrices, and $a(2) = 4$ since there are four matrices that are binary-bisymmetric, namely,

$$
\begin{bmatrix}
  1 & 1 \\
  1 & 1 \\
\end{bmatrix}, \begin{bmatrix}
  1 & 0 \\
  0 & 1 \\
\end{bmatrix}, \begin{bmatrix}
  0 & 1 \\
  1 & 0 \\
\end{bmatrix}, \text{ and } \begin{bmatrix}
  0 & 0 \\
  0 & 0 \\
\end{bmatrix}.
$$

To avoid specifying each binary bisymmetric matrix while counting, we make use of recursive properties for $a(n)$. If an $n \times n$ matrix $A$ has its first and last rows and columns removed, the remaining $(n-2) \times (n-2)$ matrix will be called the inside matrix of $A$ and will be denoted $I(A)$. Matrix $A$ will be binary bisymmetric if and only if $I(A)$ is binary bisymmetric. Hence $a(n) = 2^n \cdot a(n-2)$. For example, $a(4) = 2^4 a(2) = 2^4 (4) = 64$. The 64 matrices are displayed by their grid proxies below in which a darkened cell represents a one and an undarkened cell represents a zero.
Since $a(0) = 1$ and $a(1) = 2$, using the recursion $a(n) = 2^na(n-2)$ iteratively, we obtain $a(2) = 2^2(1) = 4$, $a(3) = 2^3(2) = 16$, $a(4) = 2^4(4) = 64$, $a(5) = 2^5(16) = 512$, $a(6) = 2^6(64) = 4096$, etc. What we get is either $a(n) = 2^n(2^{n-2}(2^{n-4}(\ldots2^2(a(0))\ldots)))$ or $a(n) = 2^n(2^{n-2}(2^{n-4}(\ldots2^3(a(1))\ldots)))$.

In other words,

$$a(n) = \begin{cases} 2^{n+n-2+\cdots+2} & \text{for even } n \\ 2^{n+n-2+\cdots+1} & \text{for odd } n \end{cases}.$$ 

Since, for even $n$, $n + n - 2 + \cdots + 2 = n(n + 2)/4$
and, for odd $n$, $n + n - 2 + \cdots + 1 = (n + 1)^2/4$, we obtain

$$a(n) = \begin{cases} 2^{\frac{n(n+2)}{4}} & \text{for even } n \\ 2^{\frac{(n+1)^2}{4}} & \text{for odd } n. \end{cases}$$

or

$$a(n) = 2^{\lfloor(n+1)^2/4\rfloor} \text{ for } n = 0, 1, 2, 3, \ldots.$$
The first terms of the sequence are generated by *Maple* software below:

```maple
> seq(2^floor((n+1)^2/4), n=0..20);
1, 2, 4, 16, 64, 512, 4096, 65536, 1048576, 33554432,
1073741824, 68719476736, 4398046511104, 562949953421312,
72057594037927936, 18446744073709551616,
4722366482869645213696, 2417851639229258349412352,
1237940039285380274899124224, 1267650600228229401496703205376,
1298074214633706907132624082305024
```

We note that a shift to the right of this sequence by one unit results in sequence A060656 in the *On-Line Encyclopedia of Integer Sequences* [http://oeis.org/A060656]. We have thus provided objects (binary bisymmetric matrices) for sequence A060656 to count.