4.1 Maximum and Minimum Values

Determine all critical numbers for the function.

1) \( f(x) = x^3 - 12x - 4 \)

Find the absolute extreme values of the function on the interval.

2) \( g(x) = -x^2 + 9x - 18, \quad 3 \leq x \leq 6 \)

3) \( f(\theta) = \sin\left(\theta + \frac{\pi}{2}\right), \quad 0 \leq \theta \leq \frac{7\pi}{4} \)

4) \( f(x) = x^{4/3}, \quad -1 \leq x \leq 8 \)

5) \( f(x) = e^x - x, \quad -1 \leq x \leq 2 \)

Find the extreme values of the function and where they occur.

6) \( y = x^3 - 3x^2 + 1 \)

Solve the problem.

7) The velocity of a particle (in \( \text{ft/s} \)) is given by \( v = t^2 - 7t + 5 \), where \( t \) is the time (in seconds) for which it has traveled. Find the time at which the velocity is at a minimum.

4.2 The Mean Value Theorem

Determine whether the function satisfies the hypotheses of the Mean Value Theorem for the given interval.

8) \( f(x) = x^{1/3}, \quad [-4, 5] \)

9) \( g(x) = x^{3/4}, \quad [0, 1] \)

Find the value or values of \( c \) that satisfy the equation \( \frac{f(b) - f(a)}{b - a} = f'(c) \) in the conclusion of the Mean Value Theorem for the function and interval.

10) \( f(x) = x^2 + 5x + 2, \quad [-3, -2] \)

11) \( f(x) = \ln(x - 1), \quad [2, 6] \)
Provide an appropriate response.

12) A trucker handed in a ticket at a toll booth showing that in 2.5 hours he had covered 217 miles on a toll road with speed limit 65 mph. The trucker was cited for speeding. Why?

4.3 How Derivatives Affect the Shape of a Graph

Find the largest open interval where the function is changing as requested.

13) Increasing f(x) = x² - 2x + 1

14) Decreasing f(x) = √(4 - x)

15) Decreasing: f(x) = \frac{x + 6}{x - 3}

Locate the critical points of the function. Then use the First Derivative Test to determine whether they correspond to local maxima or local minima.

16) f(x) = x³ - 12x + 3

Determine where the given function is concave up and where it is concave down.

17) f(x) = x√(9 - x²)

18) f(x) = x³ + 3x² - x - 24

Locate the critical points of the function. Then use the Second Derivative Test to determine (if possible) whether they correspond to local maxima or local minima.

19) f(x) = x³ - 3x² + 7

Use the graph of the function f(x) to locate the local extrema and identify the intervals where the function is concave up and concave down.

20)
4.4 Indeterminate Forms and l'Hospital's Rule

21) List the seven indeterminate forms.

Use l'Hopital's Rule to evaluate the limit.

22) \[ \lim_{x \to 5} \frac{x^2 - 7x + 10}{x - 5} \]

23) \[ \lim_{x \to 0} \frac{\cos 3x - 1}{x^2} \]

24) \[ \lim_{x \to 0} \frac{\sin 5x}{\sin x} \]

25) \[ \lim_{x \to \infty} x \sin \frac{5}{x} \]

26) \[ \lim_{x \to \infty} \left( \sqrt{x^2 + 3x - x} \right) \]

Find the limit.

27) \[ \lim_{x \to \infty} \left( \left( 1 + \frac{4}{x^5} \right)^x \right) \]

28) \[ \lim_{x \to 0} \left( e^{5/x} + 1 \right)^{x/2} \]

L'Hopital's rule does not help with the given limit. Find the limit some other way.

29) \[ \lim_{x \to 0^+} \frac{1}{\cot x \sin x} \]

30) \[ \lim_{x \to 0^+} \frac{\sec x}{\tan x} \]
31) A window is in the form of a rectangle surmounted by a semicircle. The rectangle is of clear glass, whereas the semicircle is of tinted glass that transmits only one-fourth as much light per unit area as clear glass does. The total perimeter is fixed. Find the proportions of the window that will admit the most light. Neglect the thickness of the frame.

32) At noon, ship A was 14 nautical miles due north of ship B. Ship A was sailing south at 14 knots (nautical miles per hour; a nautical mile is 2000 yards) and continued to do so all day. Ship B was sailing east at 6 knots and continued to do so all day. The visibility was 5 nautical miles. Did the ships ever sight each other? What is the minimum distance between them?

33) A rectangular field is to be enclosed on four sides with a fence. Fencing costs $6 per foot for two opposite sides, and $5 per foot for the other two sides. Find the dimensions of the field of area 870 ft² that would be the cheapest to enclose.

34) Suppose \( c(x) = x^3 - 24x^2 + 20,000x \) is the cost of manufacturing \( x \) items. Find a production level that will minimize the average cost of making \( x \) items.

35) If the price charged for a candy bar is \( p(x) \) cents, then \( x \) thousand candy bars will be sold in a certain city, where \( p(x) = 43 - \frac{x}{20} \). How many candy bars must be sold to maximize revenue?

36) Find the number of units that must be produced and sold in order to yield the maximum profit, given the following equations for revenue and cost:

\[
R(x) = 50x - 0.5x^2 \\
C(x) = 4x + 6.
\]
3.9  Related Rates

Solve the problem. Round your answer, if appropriate.

37) A company knows that the unit cost $C$ and the unit revenue $R$ from the production and sale of $x$ units are related by $C = \frac{R^2}{90,000} + 8696$. Find the rate of change of unit revenue when the unit cost is changing by $13/\text{unit}$ and the unit revenue is $4000$.

38) A wheel with radius 3 m rolls at 17 rad/s. How fast is a point on the rim of the wheel rising when the point is $\pi/3$ radians above the horizontal (and rising)? (Round your answer to one decimal place.)

39) Electrical systems are governed by Ohm’s law, which states that $V = IR$, where $V =$ voltage, $I =$ current, and $R =$ resistance. If the current in an electrical system is decreasing at a rate of $5 \text{ A/s}$ while the voltage remains constant at 12 V, at what rate is the resistance increasing (in $\Omega/\text{sec}$) when the current is 36 A? (Do not round your answer.)

40) The radius of a right circular cylinder is increasing at the rate of 9 in./sec, while the height is decreasing at the rate of 3 in./sec. At what rate is the volume of the cylinder changing when the radius is 16 in. and the height is 20 in.?

41) One airplane is approaching an airport from the north at 199 km/hr. A second airplane approaches from the east at 162 km/hr. Find the rate at which the distance between the planes changes when the southbound plane is 26 km away from the airport and the westbound plane is 24 km from the airport.
1) \(x = -2\) and \(x = 2\)

2) absolute maximum is \(\frac{9}{4}\) at \(x = \frac{9}{2}\); absolute minimum is 0 at 6 and 0 at \(x = 3\)

3) absolute maximum is 1 at \(\theta = 0\); absolute minimum is -1 at \(\theta = \pi\)

4) absolute maximum is 16 at \(x = 8\); absolute minimum is 0 at \(x = 0\)

5) absolute minimum value is 1 at \(x = 0\); absolute maximum value is \(e^2 - 2\) at \(x = 2\)

6) Local maximum at \((0, 1)\), local minimum at \((2, -3)\).

7) 3.5 sec

8) No

9) Yes

10) \(\frac{5}{2}\)

11) \(c = -\frac{4}{\ln(5)} + 1\)

12) As the trucker’s average speed was 87 mph, the Mean Value Theorem implies that the trucker must have been going that speed at least once during the trip.

13) \((1, \infty)\)

14) \((-\infty, 4)\)

15) \((-\infty, 3), (3, \infty)\)

16) local maximum: \((-2, 19)\); local minimum: \((2, -13)\)

17) Concave up on \((-3, 0)\), concave down on \((0, 3)\)

18) Concave up on \((-1, \infty)\), concave down on \((-\infty, -1)\)

19) local maximum: \((0, 7)\); local minimum \((2, 3)\)

20) Local minimum at \(x = 3\); local maximum at \(x = -3\); concave up on \((0, \infty)\); concave down on \((-\infty, 0)\)

21) 3

22) 3

23) \(\frac{9}{2}\)

24) 5

25) 5

26) \(\frac{3}{2}\)

27) 1

28) \(e^{2.5}\)

29) 1

30) \(\infty\)

31) \(\frac{\text{width}}{\text{height}} = \frac{16}{8 + 3\pi}\)

32) No. The closest they ever got to each other was 5.5 nautical miles.

33) 26.9 ft @ $6 by 32.3 ft @ $5

34) 12 items

35) 430 thousand candy bars

36) 46 units
37) $146.25/\text{unit}$
38) 25.5 m/s
39) $\frac{5}{108} \Omega/\text{sec}$
40) $4992\pi \text{ in.}^3/\text{sec}$
41) -256 km/hr