INDIFFERENCE CURVE ANALYSIS: BEYOND SIMPLIFYING ASSUMPTIONS[1]

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Abstract

This article presents the proper analysis of indifference curve systems once some of the simplifying assumptions used to teach beginning students are relaxed. First, the diminishing MRS assumption is relaxed to allow the existence of concave[4] indifference curves. Then the more-is-better assumption is relaxed. This reveals a potential source of confusion for students: usually one can find the optimum of a system by choosing the point where the budget line touches the most northeastern indifference curve, but once the more-is-better assumption is relaxed, the preference order of the indifference curves becomes the main criterion – not geographical location. Once both assumptions are removed, we study the case where too much of a good thing can become a bad thing – that is, where the consumer can encounter satiation.

Key words: Indifference curve, convex, concave, budget line

JEL Classification: B40

Introduction

Several simplifying assumptions are typically made when introducing students to indifference curve analysis,. These assumptions, although useful, give students only a partial view of the whole edifice of indifference curves. We focus on the proper analysis of indifference curve systems once some of these assumptions have been relaxed. The assumption of diminishing marginal rates of substitution (MRS) is withdrawn first to arrive at concave indifference curves. Then, the more-is-better assumption is relaxed to arrive at the case where the goods become bads in all or part of the indifference curve system. After parting with these assumptions, we present some of the consequences for the analysis of indifference curves.

In the following section, we set out what we believe is the correct indifference curve analysis. We conclude in the third section.

Indifference curve analysis

The ordinary downward sloping, convex-to-the-origin[5] indifference curve (our diagram 1) is only one small part of the indifference curve edifice. For this case, we make the artificial assumptions that more of a good is always preferred to less and that both goods in the two dimensional version of this diagram are subject to diminishing MRS. When a budget line is

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[4] Concave to the origin.

[5] Or, for short, convex indifference curves.

added, we arrive at diagram 2. In this diagram, the optimum is B, because at this point the budget line is tangent to the most preferable indifference curve it can reach.

Diagram 1: Convex Indifference Curves

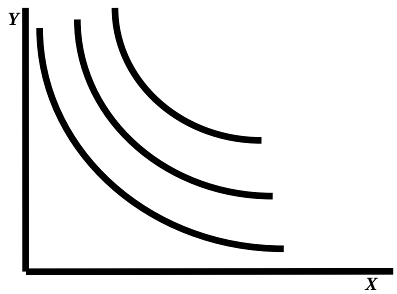
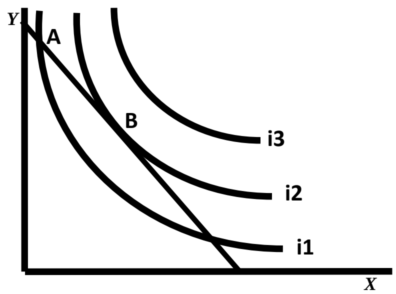
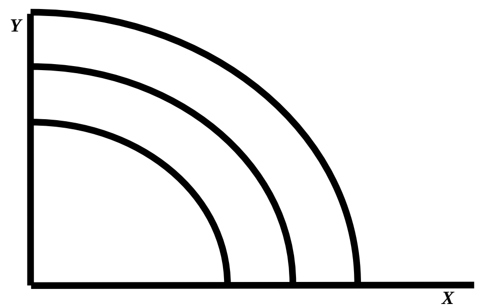


Diagram 2: Convex Indifference Curves with Budget Line



When we relax the assumption that both goods are subject to diminishing MRS, we can conceive a concave indifference curve system that looks like diagram 3.

Diagram 3: Concave Indifference Curves

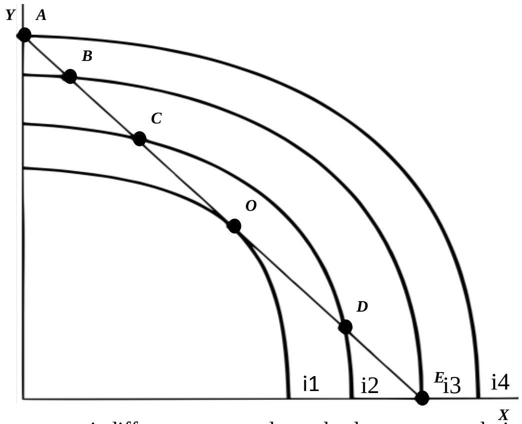


In this case, the consumer shows increasing MRS. That is, the more of X he has, the more he values an additional unit of X. Many intermediate micro texts show diagrams similar to Landsburg's (2011, 59, ex. 3.10), repeated in our diagram 4, where we label the indifference curves. Since we are still holding the more-is-better assumption, we know that i1<i2<i3<i4. Since i4 is the most preferred indifference curve, the utility maximizing point occurs at one of the axes, where the budget line intersects i4 (point A), and there is no tangency between the budget line and an indifference curve at this point. This situation is widely known as the corner solution.

Before we relax the second simplifying assumption, we want to point out a potentially confusing result for students. So far, in both convex and concave indifference curve systems, the optimum occurs where the budget line meets the highest indifference curve, (i2 in Diagram 2, and i4 in Diagram 4). As we show below, it is erroneous to assume that the optimum always occurs where the budget line touches the highest indifference curve geographically.

To exemplify this potential confusion, we will show two cases of utility functions that yield indifference curve systems like the one on diagram 4, yet have different optimums.

Diagram 4: Consumer Choice with Non-convex Indifference Curves



Non-convex indifference curves always lead to a corner solution. The consumer pictured here will choose point A, which is on the highest indifference curve.

The first case is:

$$U_1 = Ax^2 + By^2 \quad ,$$

where A and B are positive constants. In this case, the system satisfies the condition that i1<i2<i3<i4, and the corner solution is correct.

If we relax the more-is-better assumption, we can consider the case of garbage goods, or "bads," which leads us to the second utility function:

$$U_2 = -Ax^2 - By^2.$$

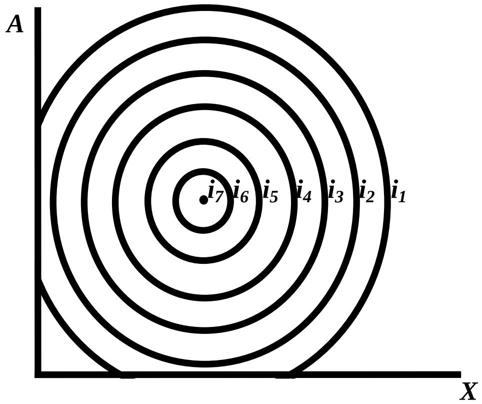
In this case, the indifference curve system looks exactly like the one for U_1, except that the ordering condition for the indifference curves is reversed. That is, i1>i2>i3>i4. The optimum occurs where the budget line is tangent to i1 (point O in Diagram 4). In this case, point A (the corner solution) is a utility minimizing point.

The purpose of these examples is to show that using a geographical location criterion for selecting an optimum is not strong enough. In fact, the optimum point cannot be determined when the ordering condition of the indifference curves is unknown.

Now that we have relaxed these simplifying assumptions, we are able to study the case where too much of a good thing can become a bad thing. Not only is marginal utility diminishing, it can also become negative; then, a good becomes a bad, or trash, or a garbage good. When these cases are incorporated into the analysis, we arrive at diagram 5 in which a family of seven circular indifference curves is depicted. We drew circular indifference curves for simplicity, but the main idea here is that a family of indifference curves is nothing else than a contour map of the two-variable utility function.

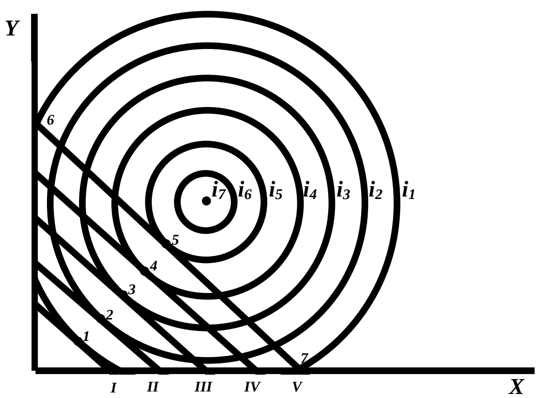
Diagram 5 represents the case where too much of a good thing becomes a bad thing. The indifference curves are labeled in order of *decreasing* desirability. That is, i7>i6>i5>i4>i3>i2>i1; location on i1 is the least preferred position. An important element shown in this diagram is that of *satiation*. If the consumer is located on i7, he cannot become more satisfied by changing his consumption.

Diagram 5: Family of Circular Indifference Curves



In Diagram 6 we illustrate the tangency points between the budget line and the indifference curves for the "ordinary" part of the indifference map. The only departure from normal practice is that we show these tangency points for the *entire*, *circular* indifference curve set,[6] not just the downward sloping convex part.

Diagram 6: Family of Indifference Curves with Budget Lines



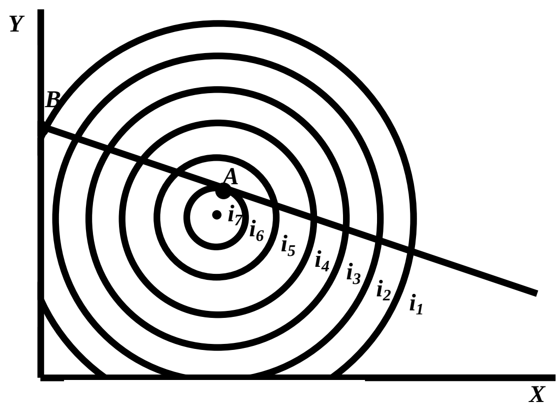
This leads us to our diagram 7, the high point of our entire analysis. We again employ a family of circular indifference curves. This time, however, the budget line is tangent to indifference curve i6 at point A and intersects indifference curve i1 at point B.

Which point is preferred by the consumer? Clearly, A>B, since A lies on indifference curve i6, B lies on i1, and i6>i1. To be sure, one can easily err, here. One might think that i1 is preferred to i6, because part of i1 lies higher that i6 geographically (up and to the right of all of i6), but this would be a grave error. The closer to the satiation point, the more preferable the indifference curve and i6 is closer to this point, i7, than is any part of i1.

6 Some of the few texts that utilize circular indifference curve families are Varian, 2006. p. 43, figure 3.7; Boulding, 1966, 605, figure 136; Mathis and Koscianski, 2002, p. 60, figure 3.13; Vickrey, 1964, p. 37, figure 6 and McCloskey, 1982, p. 27, figure 2.2. This is greatly to their credit, as most texts do not offer this crucially important aspect of indifference curves.

Let us look at this indifference curve set as if it were a three dimensional contour map, with i7 as the very tip top of the mountain. When viewed from this perspective, i6 is the next highest ridge on this mountain, and i1 is at the very bottom of the hill, at ground level.

Diagram 7: Tangency with the Budget Line



We have claimed that A is preferable to B since i6>i1. But what point will the consumer choose? If the consumer is constrained to spend his entire budget on goods X and Y, then he will choose point A. Alternatively, if the consumer is allowed to save (or destroy) part of his budget, then he will surely position himself on the very top of the mountain at i7. At i7 the consumer faces a state of satiation – additional quantities of both X and Y have now become garbage such that any additional consumption decreases utility.

Conclusion

The diminishing MRS and more-is-better assumptions are useful to introduce students to indifference curves without overwhelming them with details. Relaxing these assumptions, however, allows a richer analysis and understanding of the subject. Relaxing the diminishing MRS assumption makes it possible to study a concave indifference curve, where the optimum is given by the corner solution. If, in addition, the more-is-better assumption is also relaxed, the corner solution does not necessarily apply. In this case, using a geographical location criterion for selecting an optimum (selecting the point where the budget line meets the most northeastern indifference curve) is not accurate, because the preference ordering of the indifference curves must be considered. Withdrawing these assumptions allows us to study the case where too much of a good thing becomes a bad thing. In this case, we find circular families of indifference curves. Their treatment is similar to the traditional indifference curves in that the optimum lies on the most preferable indifference curve within the budget constraint. In contrast to the traditional analysis, however, this may not be a point on the budget line. When satiation is present, the optimum can be a point below the budget line.

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