

ON TEACHING PRODUCTION THEORY: INTEGRATING SHORT-RUN AND LONG-RUN ANALYSES

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Abstract

The microeconomic theory of production is typically presented to undergraduates in a way that leaves many students with a disjointed view of the subject. The short run is described as one kind of world, in which the firm pursues one set of objectives, commonly identified as “optimal factor employment.” The long run, on the other hand, is depicted as a very different kind of world, in which the firm pursues a different set of objectives, known as “optimal factor combination.” The purpose of this essay is to bring together familiar diagrammatic tools in a way that allows instructors to integrate short-run and long-run principles of factor employment.

JEL Classification: A20, A22, A23, D21, D24

Dis-Integration

Microeconomics textbooks define the short run as the time period in which at least one factor of production (usually capital) is fixed, and the long run as the time necessary for all factors to be variable. The standard presentation discusses the short run in terms of *optimal factor employment*, in which the firm employs some variable factor (usually labor) to the level at which the factor’s marginal revenue product is equal to its marginal factor cost ($mrp = mfc$). The long-run analysis, on the other hand, assumes that all factors are variable, and the firm pursues the *optimal factor combination*, which is achieved when the ratio of factor prices equals the ratio of marginal products ($mfc_a/mfc_b = mp_a/mp_b$). Textbooks often offer little discussion of how the short-run and long-run optimality conditions are related – and even less about how the firm gets from one “run” to the other.²

A more integrative analysis would describe the *process* by which the firm adjusts employment, through time, in accordance with both factor employment and factor combination criteria. The purpose of this essay is to develop such an analysis, using familiar diagrammatic tools which are left unconnected in the typical pedagogical approach. First, we review the optimal factor employment and factor combination conditions, with emphasis on the logical relationships between these conditions in the context of profit maximization. Next, we consider the ways in which different inputs interact in the production process – specifically, issues of

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² We examined numerous intermediate level microeconomics and managerial economics textbooks. We do not identify them, since we are not attempting to text books, nor do we wish to imply any qualitative judgments concerning the books in our sample.

substitutability and complementarity. We then present an integrated analysis using familiar diagrammatic tools.

Optimum Conditions of Factor Employment and Factor Combination

We may describe a production process generally as $q = f(l, k)$ where l is labor and k is capital. To simplify the analysis, assume the firm is a price taker in both input and output markets, so that marginal revenue is the same as product price (P), and marginal factor costs are identical to the wage (w) and the rental fee (r) for labor and capital, respectively. Assume also that the firm is operating where factor marginal products are positive but declining. The marginal products of labor and capital are:

$$\begin{aligned} mp_k &= \partial q / \partial k \text{ (labor held constant)} \\ mp_l &= \partial q / \partial l \text{ (capital held constant)} \end{aligned}$$

And the firm is operating such that:

$$\begin{aligned} \partial q / \partial k &> 0 & \partial(\partial q / \partial k) / \partial k &< 0 \\ \partial q / \partial l &> 0 & \partial(\partial q / \partial l) / \partial l &< 0 \end{aligned}$$

The value of marginal product (vmp) of each factor is:

$$\begin{aligned} vmp_k &= P(mp_k) \\ vmp_l &= P(mp_l) \end{aligned}$$

Total cost is defined as:

$$C = rk(q) + wl(q)$$

And the marginal cost associated with each factor³ is:

$$\begin{aligned} MC_l &= w/mp_l \\ MC_k &= r/mp_k \end{aligned} \qquad \text{Marginal Costs} \qquad (1)$$

The familiar optimal factor employment condition states that a factor should be employed up to the point at which the value of its marginal product is equal to its own marginal cost:

$$\begin{aligned} vmp_l &= w \\ vmp_k &= r \end{aligned} \qquad \text{Optimal Factor Employment} \qquad (2)$$

As a short-run condition, the optimal factor employment of each factor is meaningful on its own terms without reference to the other factor, assumed to be fixed.

³ For labor: $MC_l = \Delta C / \Delta l = w\Delta l / \Delta q = w / (\Delta q / \Delta l) = w / mp_l$. Likewise, $MC_k = r / mp_k$.

The long run is commonly defined as the time frame within which all factors are variable. The optimal combination of factors when both factors are variable is achieved when the marginal product per dollar spent is the same for the two inputs:

$$mp_l/w = mp_k/r \quad \text{Optimal Factor Combination} \quad (3)$$

The short run and long run conditions bear a relationship that is seldom made explicit in intermediate-level microeconomics textbook presentations. In the context of profit maximization ($P = MC$), condition (2) is equivalent to:

$$\begin{aligned} w &= P(mp_l) \\ \text{and} \\ r &= P(mp_k). \end{aligned}$$

Thus $w/mp_l = P$ and $r/mp_k = P$. If $MC = P$, then $w = MC(mp_l)$ and $r = MC(mp_k)$. Therefore, when the firm is maximizing profit, (2) implies:

$$w/mp_l = MC = r/mp_k \quad (4)$$

When the firm is maximizing profit, the marginal cost of increasing output must be the same, whether the increase comes from employing more capital, or more labor. Thus the two marginal costs defined in (1) are equal to each other.

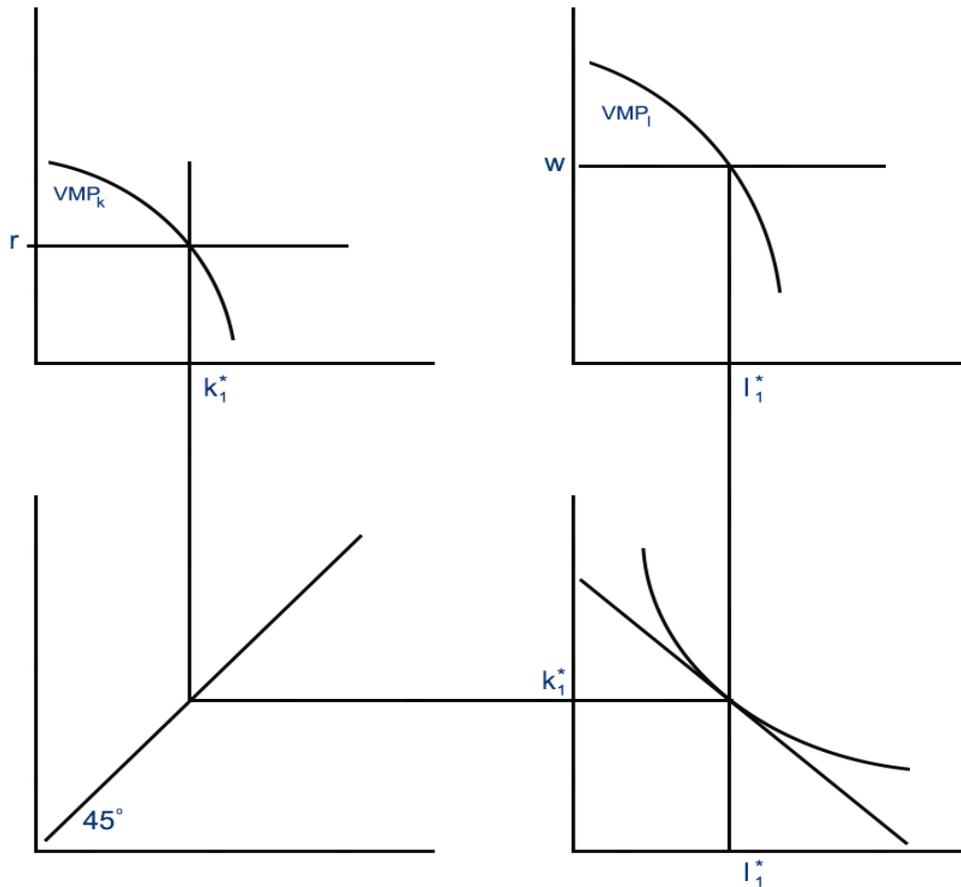
It follows from (4) that $mp_l/w = 1/MC = mp_k/r$. Multiplying through by P yields $vmp_l/w = P/MC = vmp_k/r$. And if $P = MC$, then $vmp_l/w = 1 = vmp_k/r$. This is equivalent to the optimal factor combination condition (3). Furthermore, when (4) pertains, condition (2) is achieved for both factors simultaneously. If (2) were not met for both, then the firm could increase profit by expanding employment of either factor with a marginal cost lower than price, and reducing employment of the factor with a marginal cost greater than price. If, for instance, $r < vmp_k$ then $r < P(mp_k)$ and $MC_k = r/mp_k < P$. Likewise, if $w > vmp_l$ then $w > P(mp_l)$ and $MC_l = w/mp_l > P$. In the first instance, the firm should increase employment of capital (because $P > MC_k$) and in the second instance, reduce employment of labor ($P < MC_l$).

The upshot is that when short-run optimality conditions (2) are met simultaneously for both factors, the long-run condition (3) must obtain as well. Note that in the integrative process-oriented analysis developed here, successive short-run factor adjustments are seen as *yielding* the long-run optimum factor combination. This relationship between short-run and long-run conditions is represented diagrammatically in Figure 1. The top half of the figure illustrates optimal factor employment conditions, while the bottom half depicts the optimal factor combination. This “full equilibrium” condition, i.e., the condition in which both short-run and long-run equilibria obtain, will serve as the starting point for analyses that follow.

Of course, it is unlikely in the real world that an optimal short-run equilibrium would coincide with long-run equilibrium; if so, it is a happy coincidence. The purpose in presenting simultaneous equilibria in both the short run and the long run is to establish an analytical starting point for discussion. The discussion proceeds by assuming general equilibrium, introducing a disequilibrating disturbance, and following the firm’s decisions, which lead – analytically speaking – to a new equilibrium. This approach (from equilibrium to disequilibrium to a new

equilibrium) is the standard analytic model in much of microeconomic and macroeconomic exposition.

Figure 1



Substitutability, Complementarity and the Shape of the Isoquant: a Necessary Digression

The vmp_k and vmp_l curves in Figure 1 are both downward sloping due to diminishing marginal product, and the isoquant is convex to the origin in part for the same reason. But employment of *both* factors varies along an isoquant, which raises the question of complementarity, i.e., the possibility that the level of employment of one factor might affect the marginal product of the other. Complementarity can be defined generally as:

$$\partial(\partial q/\partial k)\partial l = \partial\partial q/(\partial l\partial k) = \partial(\partial q/\partial l)\partial k \tag{5}$$

And the possible values of (5) define the following relationships:

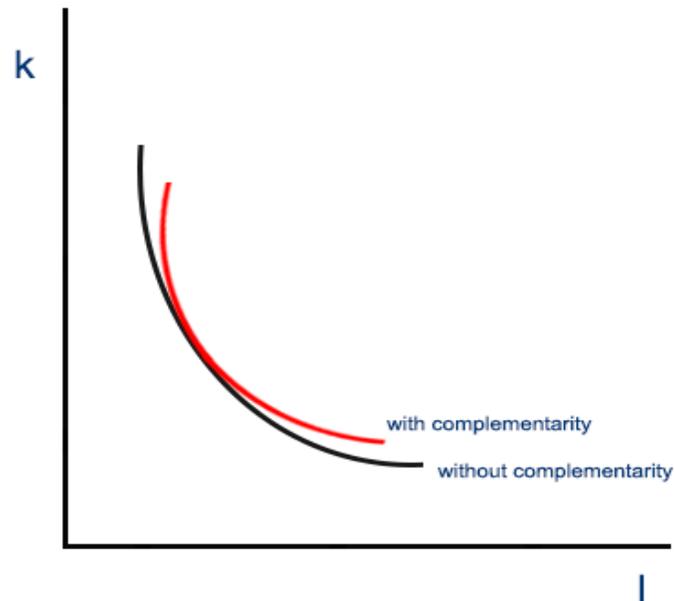
$$\partial\partial q/(\partial l\partial k) > 0 \quad \text{Complementarity} \tag{6}$$

$$\partial\partial q/(\partial l\partial k) < 0 \quad \text{Anti-complementarity} \tag{7}$$

$$\partial\partial q/(\partial l\partial k) = 0 \quad \text{Independence} \tag{8}$$

Diminishing marginal product alone implies a convex isoquant for inputs, but complementarity (6) augments the effect of diminishing marginal returns – the marginal rate of technical substitution (MRTS) changes more pronouncedly as factors are substituted along an isoquant. Figure 2 illustrates the effect of complementarity.

Figure 2

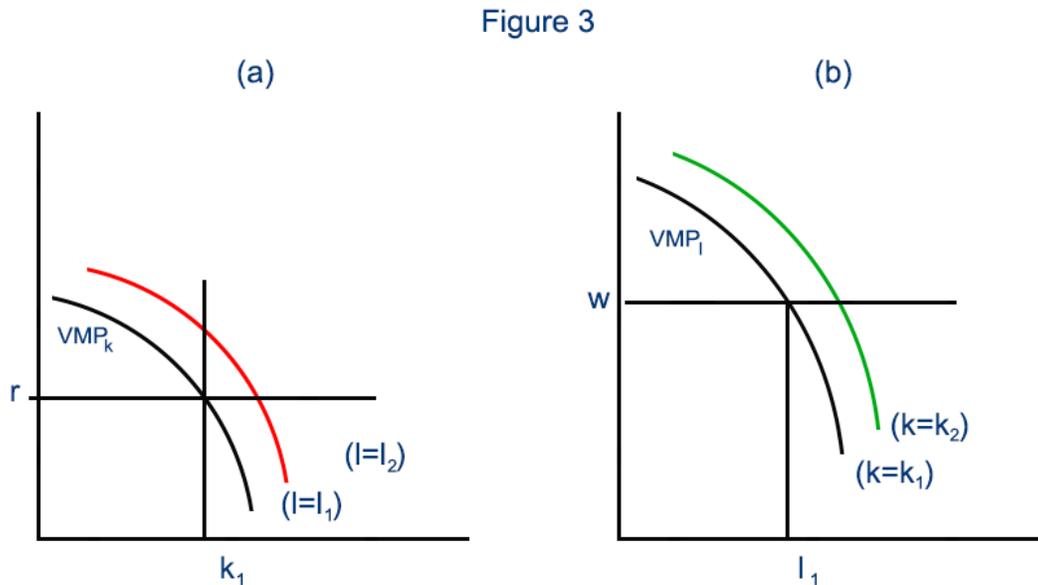


We may assume, as a practical matter, that complementarity between labor and capital exists as described in (6). For this reason, and in the interest of space, we limit subsequent analysis to this case.⁴

We have discussed complementarity in the long run context of the isoquant, but it is necessary for our purposes to depict complementarity in the short run context of optimal factor employment. Figure 3 presents the case where labor and capital are complementary: additional

⁴ Anti-complementarity (7) has the opposite effect on MRTS, and a concave isoquant would result if anti-complementarity were the dominant effect. A straight-line isoquant would result if neither factor exhibited diminishing marginal product, or if diminishing marginal product and anti-complementarity exactly offset each other. Production with fixed-input proportions is an example of extreme complementarity: the marginal product of one input is zero without a proportionate increase in employment of the other input. Both concave and straight-line isoquant curves imply corner solutions; factor substitution at the margin is technologically possible, but economically meaningless. With fixed-input production, neither technological nor economic substitution is meaningful. Independence between input factors (8) does not mean that factor substitution is economically meaningless, only that diminishing marginal returns associated with one factor is neither enhanced nor offset by changes in the level of employment of the other factor.

labor makes additional capital more productive, and vice-versa. Diagrammatically, a shift in the vmp_k curve occurs in Figure 3(a) when additional labor is employed, and vice-versa in Figure 3(b).



Discussion

We now illustrate how the integrative diagrammatic approach can be used to analyze the firm's response to changes in factor prices (wages or rental fees), a change in technology, and a change in product price. These exercises can be incorporated into the usual textbook treatment of production theory with minimal additional space or discussion.

In each case, the analysis begins with the firm in equilibrium with respect to factor employment. Then we introduce some disequilibrating change – an improvement in technology, a change in wages, an increase or decrease in the market price of the product. The firm then undertakes a series of adjustments in factor employment, arriving finally at a new equilibrium, where no further adjustments are motivated.⁵

While the short run is often defined as the time frame in which at least one variable is fixed (and the long run as the time frame sufficiently lengthy for all variables to change), another option, however, is to alter the number of factors that may be allowed to vary, irrespective of time frame considerations. The short run is then defined as limiting the number of variable factors and the long run as allowing more (or all) factors to vary. The first option defines the length of the run in terms of a specified *time frame*, while the second defines the length of the run in terms of a specified *analytical framework*. In fact, the first approach is equivalent to the second if lengthening the time frame serves merely to bring more variables into play. However,

⁵ Fritz Machlup [2, 4] describes a standard analytic model of economics as follows.

- Step 1. Initial Position: “equilibrium,” i.e., “Everything could go on as it is.”
- Step 2. Disequilibrating Change: “new datum,” i.e., “Something happens.”
- Step 3. Adjusting Changes: “reactions,” i.e., “Things must adjust themselves.”
- Step 4. Final Position: “new equilibrium,” i.e., “The situation calls for no further adjustments.” The exercises discussed here proceed along the lines described by Machlup.

the second option does not necessarily imply a longer time frame for adjustment. As Machlup [2, 4] points out, the opposite could be the case, because the firm has greater flexibility, the more factors it can vary.

The analysis here limits the list of factors to labor and capital, and allows both to vary symmetrically with regard to time. By this we mean that neither factor is assumed to be necessarily any more or less “fixed” than the other in time. Specifically, we do not assume that only labor varies in the short run, while capital adjustments occur later on.⁶ Our approach recognizes that the firm is likely to find it practical to vary employment of some factors, but not others, at any given point in time – for any number of reasons, both technical and contractual. The firm thus undertakes a series of steps by which the employment level of one factor changes, then the employment level of the other, and so on, until no further adjustments are motivated.

This process is sequential in a sense, but does not include a formal time variable. While the firm proceeds through a series of adjustments that takes it from disequilibrium, *at some point in time*, to a new equilibrium *at a later point in time*, we do not partition the process into distinct short-run and long-run temporal phases. The long-run equilibrium condition, as described in equation (3), is the *result* of this process, not a different goal appropriate to a different temporal realm.

Example A: A Decrease in Labor Cost

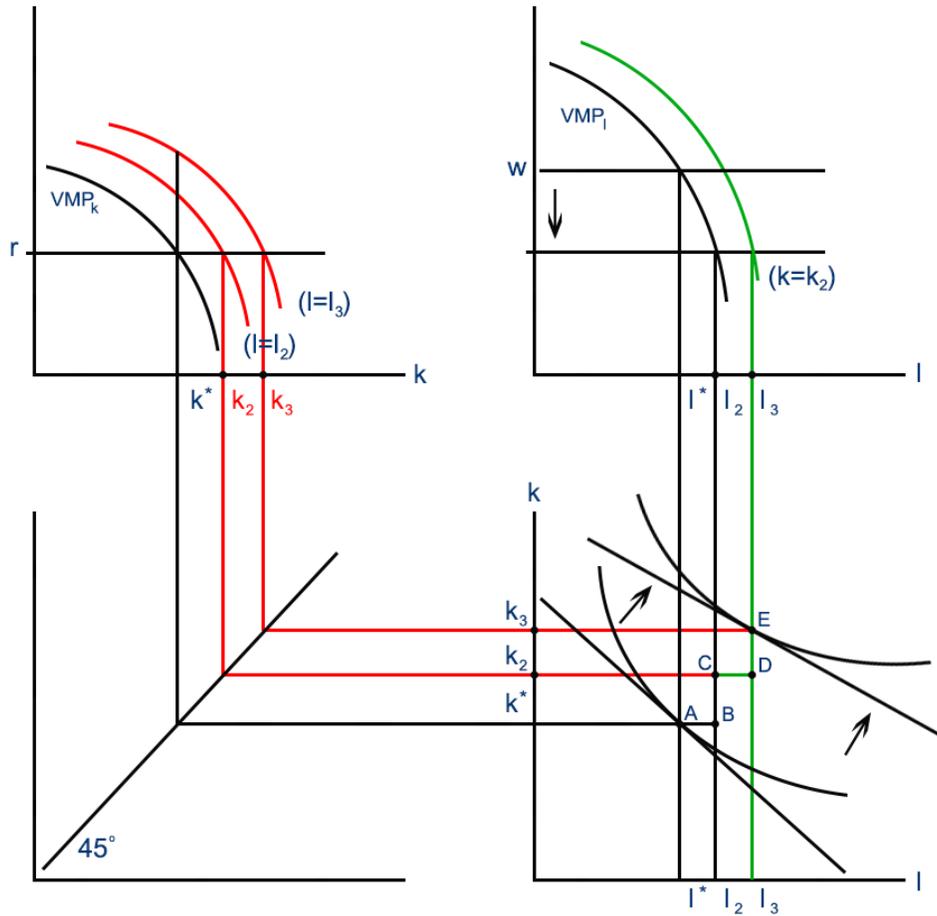
A decrease in the wage, w , motivates the firm to hire more labor, to employment level l_2 in Figure 4. The isocost line becomes less steep to reflect the change in the relative prices of capital and labor, and shifts out to pass through point B, the new optimal level of labor employment.⁷

Employment of capital remains unchanged at first; but as labor employment increases, the marginal product of capital is enhanced due to complementarity: the vmp_k shifts upward to $vmp_k (l=l_2)$. Thus (2) is no longer satisfied for both factors, because $vmp_k > r$ after labor is added. Employment of capital increases to k_2 and the firm moves to point C. But this increase in capital subsequently increases the marginal product of labor (again due to complementarity). The vmp_l curve shifts upward, employment increases to level l_3 , and the firm reaches point D. Additional labor further strengthens the marginal product of capital, and so on.

⁶ This development is “symmetrical” in that either variable, labor or capital, might be variable in a shorter period of time

⁷ Note that the isocost line is not exactly analogous to the budget constraint of utility maximization theory. The firm *chooses* the level of total spending; it is not constrained in the way that the consumer’s budget is constrained by income. At point B, total spending might be greater or less than before, depending on total expenditures on labor at this point.

Figure 4

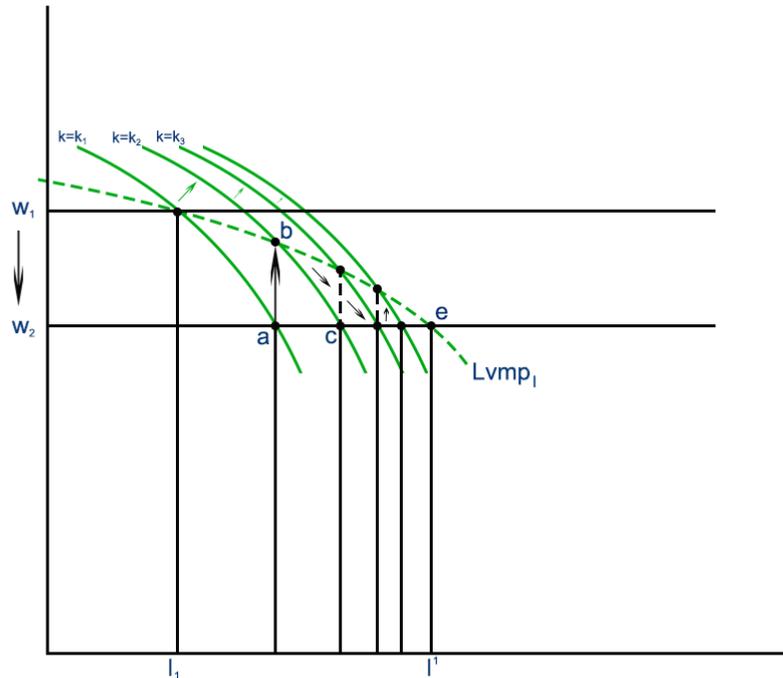


This process cannot go on forever, of course. To better see how the process plays itself out (and converges on long-run equilibrium), consider Figure 5, which presents the factor employment condition for labor.

When wages fall, labor employment increases, and the firm moves down along the initial vmp_l curve. If there were no complementarity with capital, the initial adjustment to (a) would be the full adjustment. But with complementarity, the firm is motivated to hire additional capital, which, in turn, causes the marginal product of labor to increase, thus shifting the vmp_l to the right. The firm moves from (a) to (b) then to (c). As labor employment expands further, this “reverberation”⁸ process continues until the firm arrives at some point (e), where (2) is met for both labor and capital.

⁸ We borrow the term from Hershleifer and Glazer [1, 305].

Figure 5



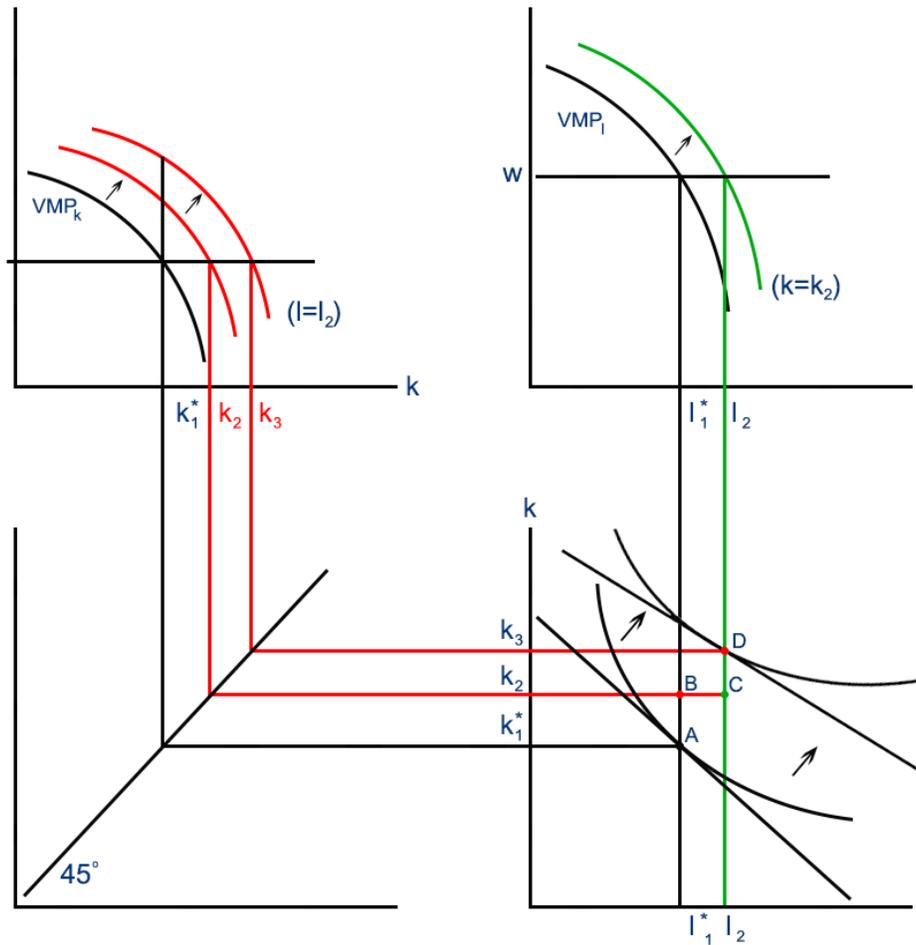
There emerges a “long-run” vmp_l curve ($lvmp_l$) which is downward sloping, but less steep than any given short-run vmp_l curve, due to the influence of complementarity. The process eventually establishes an equilibrium level of factor employment – given that the $lvmp_l$ is downward sloping. In other words, the effect of complementarity cannot be so strong as to overcompensate for diminishing returns. Thus point (b) cannot exceed the original point at which $vmp_l = w_1$. If complementarity were of such overwhelming magnitude, the $lvmp_l$ would diverge from w_2 ; the process would not lead to a determinate level of factor employment. A similar process occurs with capital: the $lvmp_k$ converges on r to establish an equilibrium level of capital employment.

To summarize, points (b), (c) and (d) in Figure 5 are positions to which the firm is driven by the optimal factor employment condition. But each adjustment of one factor throws the condition out of equilibrium for the *other* factor. Only when the firm reaches point (e) are both labor and capital in equilibrium. Here the $lvmp_l$ and $lvmp_k$ curves have converged on w_2 and r , respectively, to determine equilibrium employment levels, and (2) is satisfied for both factors. The factor combination condition (3) must also be met, and the process of adjustment reaches its completion, i.e., simultaneous short-run and long-run equilibria.

Example B: An Improvement in Technology

Consider an improvement in technology that increases the marginal product of capital. The vmp_k shifts, and the factor employment condition motivates an increase in capital employment to k_2 in Figure 6.

Figure 6

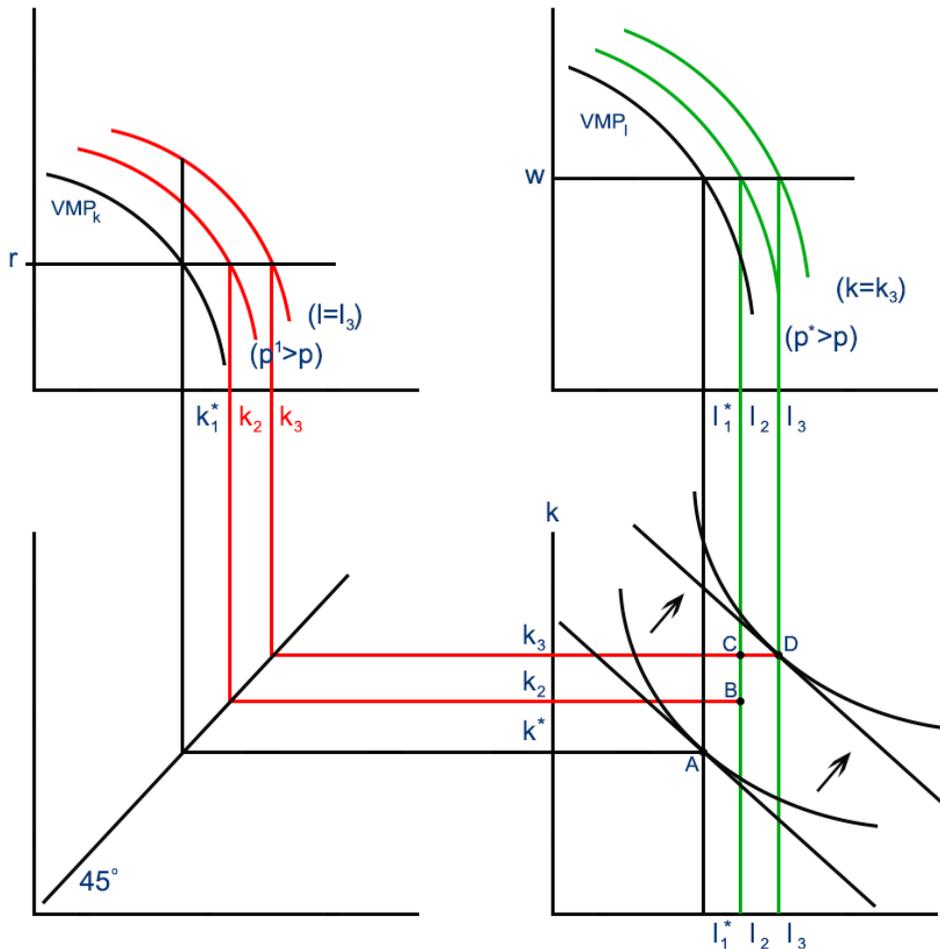


The firm moves first to point B. With the increase in capital employed, the marginal product of labor is enhanced. Thus the vmp_l shifts, labor employment goes to l_2 , and the firm reaches point C. At employment level l_2 , the marginal product of capital is enhanced, and capital employment goes to k_3 . This reverberation process continues until (2) and (4) are simultaneously satisfied (point D). That is, the process brings the firm along the $lvmp_l$ and $lvmp_k$ curves, as depicted for labor in Figure 5, to points which satisfy (2) and (3).

Example C: An Increase in Product Price

As a final exercise, consider the effects of an increase in product price. Both the vmp_k and vmp_l increase, leading to greater levels of employment of each. In Figure 7, employment levels go to k_2 and l_2 , and the firm is taken to point B.

Figure 7



Complementarity further lifts the vmp_k and vmp_l curves, and the process described above leads to point C and eventually to point D.

Summary and Conclusions

Students of microeconomics are typically presented a theory of production that is disjointed with regard to the firm's behavior in the short run as compared with the long run. The short run is described as one kind of world, in which the firm pursues one set of objectives, commonly identified as "optimal factor employment." The long run, on the other hand, is depicted as a very different kind of world, in which the firm pursues a different set of objectives, identified as "optimal factor combination." Students may be forgiven if they fail to see how one world relates to the other; textbooks should not model the long run and the short run as different worlds. This essay is an attempt to develop an alternative, more integrated, approach.

The fact is that firms operate in the present with an eye to the future. In the real world of time and circumstance, firms pursue a series of adjustments in an attempt to be efficient and earn profits. These adjustments are adequately described as pursuing optimal factor employment,

inasmuch as the firm will ordinarily find it feasible to vary some inputs, but not others, at any given point in time.

What, then, of the long run optimal combination of factors? In the approach taken here, it emerges as the *product* of actions taken in pursuit of optimal factor employment. These actions play out over time, of course, but it causes confusion among students to partition the firm's actions into separate, and ultimately artificial, temporal realms.

Unfortunately, the typical microeconomics textbook treatment of production theory does both: it partitions *and* it confuses. We hope this discussion will help to change that.

References

1. Hirshleifer, Jack and Amihai Glazer. *Price Theory and Applications*. Englewood Cliffs: Prentice-Hall, 1992.
2. Malchup, Fritz, "Misplaced Concreteness and Disguised Politics." *The Economic Journal*, Vol. LXVIII, March 1958, 1-24.