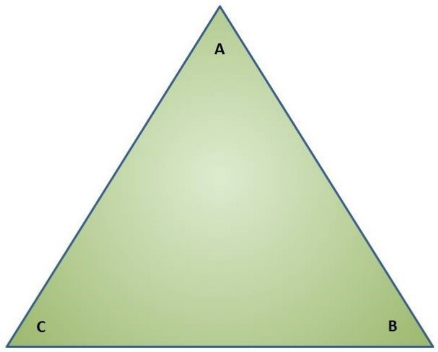
Consider a two or three dimensional object. A *symmetry* of this object is a rigid motion that maps the object onto itself (in geometry, this type of mapping is called an *isometry*). No folding, cutting, or other changes to the object are made when creating a symmetry of the object; rather, the entire object is flipped or rotated is such a way that the object in its new position occupies exactly the same space as the object in its original position.

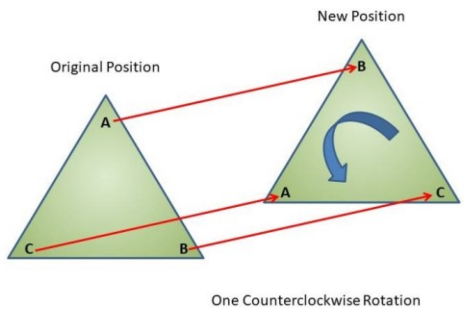
Consider the equilateral triangle shown below.



Problem 1. In the diagram above, each vertex of the triangle is labelled. Are these labels enough to keep track of how flips and rotations are affecting the triangle? Explain your reasoning.

If we let $= \{A, B, C\}$, then we can express symmetries of the triangle as members of the set $[X \to X]$. For example, if we let R represent one rotation of the triangle in the *counterclockwise* direction through a $120°$ angle, then we can write

$$R : \begin{pmatrix} A & B & C \\ B & C & A \end{pmatrix}$$

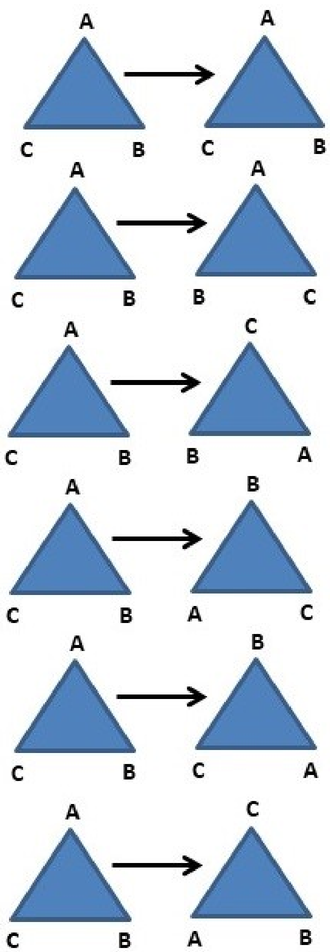


Note that we are treating the vertex labels of the initial triangle as the domain of the function.

Problem 2. Let RR represent one rotation of the triangle in the counterclockwise direction through a $240°$ angle, and let F represent one flip of the triangle about the vertical line bisecting the top angle. (The flip axis remains fixed and does not move with the triangle.) Write both RR and F as members of the set $[X \to X]$. Use tabular notation.

Problem 3. Suppose that x and y represent transformations of the triangle and let xy represent the act of performing transformation x after performing transformation y.

Part (a). Represent each of the symmetries below as combinations of the transformations R and F. (For example, $F(RR)$ would represent the symmetry obtained by performing the transformation R, followed by R, and then followed by F.) There will be multiple ways to represent each symmetry.



Problem 4. What should it mean to say that two symmetries of the equilateral triangle are *equivalent*?

Problem 5. How many non-equivalent symmetries of the equilateral triangle are there? Explain your reasoning.

Problem 6. Use your information in Problem 3 to fill in the table below. For symmetries x and y, assume that $x * y$ means "perform symmetry x after performing symmetry y."

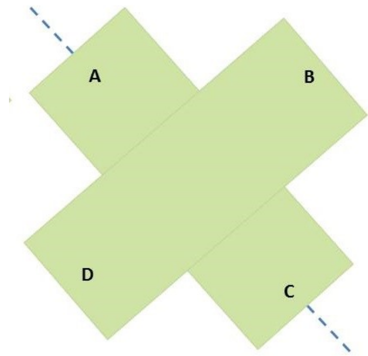
$*$	RRR	RR	R	F	FR	$F(RR)$
RRR						
RR						
R						
F						
FR						
$F(RR)$						

Problem 7. Write the transformation RF as a member of the set $[X \to X]$ in tabular notation. Is there a way to write the function representation for RF as a composition of the function representation for R and F?

Problem 8. Let $S_\Delta = \{R,\ RR,\ RRR,\ F,\ FR,\ F(RR)\}$. Explain why the rule * defined by the table above is an associative binary operation on the set S_Δ.

Homework.

Consider the cross shown in the diagram below.



Let R represent one counterclockwise rotation of the cross through a 90° angle, and let F represent one flip of the cross about the dashed line shown. (The dashed line remains fixed and does not move with the figure.)

Problem 1. There are eight non-equivalent symmetries of this cross. Let

$$S_{\times} = \{R, RR, RRR, RRRR, F, FR, F(RR), (FR)(RR)\}$$

Part (a). Let $X = \{A, B, C, D\}$. Represent each of the symmetries of the cross as a member of the family $[X \to X]$.

Part (b). Define a binary rule * on the set $S_{\times}$ by letting $x * y$ represent the symmetry obtained by performing symmetry x after performing symmetry y. Fill in the table below.

$*$	$RRRR$	RRR	RR	R	F	FR	$F(RR)$	$(FR)(RR)$
$RRRR$								
RRR								
RR								
R								
F								
FR								
$F(RR)$								
$(FR)(RR)$								

Problem 2. Compose the function representations for R and F (in an appropriate way) and compare the result to the function representation you created for the symmetry $(FR)(RR)$.

Problem 3. Explain why the binary rule * defines a binary operation on the set $S_{\times}$. Is this operation associative? Carefully explain your answer.

Problem 4. Is the binary operation * commutative? Justify your answer.