ALGEBRA CONCEPT ASSESSMENT REVIEW

1. Place the epxression $\dfrac{x^2 - x}{x(x-1)}$ in lowest terms. Use the exclusion rule when necessary.

   Solution. Observe that

$$\frac{x^2 - x}{x(x-1)} = \frac{x\,(x-1)}{x(x-1)} = 1 \qquad (x \neq 0 \text{ and } x \neq 1)$$

2. Place the expression $\dfrac{\sqrt[4]{uv^4}u}{3u^3}$ in lowest terms. Use the exclusion rule when necessary.

   Solution. Observe that

$$\begin{aligned}
\frac{\sqrt[4]{uv^4}u}{3u^3} &= \frac{u^{1/4}\left(v^4\right)^{1/4}u^1}{3u^3} \\
&= \left(\frac{1}{3}\right)\left(\frac{u^{1/4}u^1}{u^3}\right)\left(\frac{\left(v^4\right)^{1/4}}{1}\right) \\
&= \left(\frac{1}{3}\right)\left(\frac{u^{5/4}}{u^3}\right)\left(\frac{|v|}{1}\right) \\
&= \frac{|v|}{3u^{7/4}}
\end{aligned}$$

3. Write the expression $\dfrac{u}{u+4} + \dfrac{4}{u-4}$ as a single fraction. Your final answer should contain no common factors.

   Solution. Observe that

$$\begin{aligned}
\frac{u}{u+4} + \frac{4}{u-4} &= \left(\frac{u}{u+4}\right)\left[\frac{u-4}{u-4}\right] + \left(\frac{4}{u-4}\right)\left[\frac{u+4}{u+4}\right] \\
&= \frac{u(u-4) + 4(u+4)}{(u+4)(u-4)} \\
&= \frac{u^2 - 4u + 4u + 16}{u^2 - 16} \\
&= \frac{u^2 + 16}{u^2 - 16}
\end{aligned}$$

4. Use the method of completing the square to solve the equation $4t^2 - 16t = 9$.

   Solution. Observe that

$$\begin{aligned}
4t^2 - 16t = 9 &\implies 4\left[t^2 - 4t\right] = 9 \qquad B = -4 \quad \frac{B}{2} = -2 \quad \frac{B^2}{4} = 1 \\
&\implies 4\left[(t-2)^2 - 4\right] = 9 \\
&\implies (t-2)^2 - 4 = \frac{9}{4} \\
&\implies (t-2)^2 = \frac{25}{4} \\
&\implies t - 2 = \pm\sqrt{\frac{25}{4}} \\
&\implies t = 2 \pm \frac{5}{2}
\end{aligned}$$

5. Use the Distributive Rule to expand the product $(3-2y)^2(y-t)$.

Solution. Observe that

$$\begin{aligned}(3-2y)^2(y-t) &= (3-2y)(3-2y)(y-t)\\ &= [(3)(3)+(3)(-2y)+(-2y)(3)+(-2y)(-2y)](y-t)\\ &= \left[9-12y+4y^2\right](y-t)\\ &= (9)(y)+(9)(-t)+(-12y)(y)+(-12y)(-t)+\left(4y^2\right)(y)+\left(4y^2\right)(-t)\\ &= 4y^3-12y^2-4y^2t+12yt-9t+9y\end{aligned}$$

6. Solve the equation $\frac{9x}{4}+\frac{y^2}{6}=2x-3$ for the variable x.

Solution. Observe that

$$\begin{aligned}\frac{9x}{4}+\frac{y^2}{6}=2x-3 &\implies \frac{y^2}{6}+3=2x-\frac{9x}{4}\\ &\implies \frac{y^2+18}{6}=\frac{8x-9x}{4}\\ &\implies \frac{y^2+18}{6}=-\frac{x}{4}\\ &\implies -\frac{2y^2+36}{3}=x\end{aligned}$$

7. Solve the equation $2=\frac{1-y}{y+1}$ for the variable y.

Solution. Observe that

$$\begin{aligned}2=\frac{1-y}{y+1} &\implies 2(y+1)=1-y\\ &\implies 2y+2=1-y\\ &\implies 3y=-1\\ &\implies y=-\frac{1}{3}\end{aligned}$$

8. Solve the equation $4w^2+3w-1=0$ for the variable w.

Solution. Observe that

$$\begin{aligned}4w^2+3w-1=0 &\implies w=-\frac{3}{2(4)}\pm\frac{\sqrt{9-4(4)(-1)}}{2(4)}\\ &\implies w=-\frac{3}{8}\pm\frac{\sqrt{9+16}}{8}\\ &\implies w=-\frac{3}{8}\pm\frac{5}{8}\end{aligned}$$

9. Solve the equation $\frac{4}{b}+\frac{1}{b+1}=1$ for the variable b.

Solution. Observe that

$$\begin{aligned}
\frac{4}{b} + \frac{1}{b+1} = 1 \quad &\Longrightarrow \quad \left(\frac{4}{b}\right)\left[\frac{b+1}{b+1}\right] + \left(\frac{1}{b+1}\right)\left[\frac{b}{b}\right] = 1 \\
&\Longrightarrow \quad \frac{4(b+1)+b}{b(b+1)} = 1 \\
&\Longrightarrow \quad \frac{5b+4}{b(b+1)} = 1 \\
&\Longrightarrow \quad 5b+4 = b^2 + b \\
&\Longrightarrow \quad 0 = b^2 - 4b - 4 \\
&\Longrightarrow \quad b = \frac{4}{2(1)} \pm \frac{\sqrt{16-4(-4)(1)}}{2(1)} \\
&\Longrightarrow \quad b = 2 \pm \frac{\sqrt{32}}{2} \\
&\Longrightarrow \quad b = 2 \pm \sqrt{8}
\end{aligned}$$

10. Find the solution set for the inequality $3z - 5 < z + 1$. Write your answer in interval notation.

Solution. Observe that

$$3z - 5 < z + 1 \Longrightarrow 4z < 6 \Longrightarrow z < \frac{3}{2}$$

The solution set will be the ray $\left(-\infty, \frac{3}{2}\right)$.