ALGEBRA REVIEW
UNIT 2 --- Solving Linear Equations

We “solve” an equation for a given expression by using arithmetic operations to rewrite the equation so that the given expression appears by itself on one side of the new equation. There are some simple rules that must always be followed when solving an equation.

- *Solving an equation can only be accomplished using the arithmetic operations of addition, subtraction, multiplication, division, raising to a power, or taking a root.**

- *An arithmetic operation cannot be applied to just a portion of an expression --- it must be applied to the entire expression.*

- *If an arithmetic operation is applied to one side of an equation, it must also be applied to the other side of the equation.*

The following example shows how the process of solving an equation looks when it is written down carefully.

** There are other “approved” techniques that can be used, but we will not need them in these notes.*

EXAMPLE 1. Solve the equation $4x + 7y = 2y + 1$ for the variable y.

The goal is to use arithmetic operations to rewrite this equation so that y appears by itself on one side. Whatever operations we use must be applied to BOTH sides of the equation.

$$4x + 7y = 2y + 1 \implies (4x + 7y) - 1 = (2y + 1) - 1$$ ***(Subtract 1 from both sides.)***

$$\implies 4x + 7y - 1 = 2y + (1 - 1)$$ ***(Combine like terms.)***

$$\implies 4x + 7y - 1 = 2y + 0$$ ***(Simplify.)***

$$\implies (4x + 7y - 1) - 7y = 2y - 7y$$ ***(Subtract $7y$ from both sides.)***

$$\implies 4x + (7 - 7)y - 1 = (2 - 7)y$$ ***(Combine like terms.)***

$$\implies 4x + 0y - 1 = -5y$$ ***(Simplify.)***

$$\implies \frac{4x-1}{-5} = \frac{-5y}{-5}$$ ***(Divide both sides by -5.)***

$$\implies -\frac{4x-1}{5} = y$$ ***(Simplify.)***

We use an implication symbol to connect manipulations when we are working with both sides of an equation at the same time.

In the previous example, notice how we line up each step of the solution process on a separate line. Off to one side, we wrote down the arithmetic step, or the simplifying step that was used to obtain each equation. We performed only one step on each line.

The "implication arrow" is used only when we are working with both sides of an equation at the same time. When it appears, the arrow means "this equation is related to the previous equation."

You **do not** use the implication arrow if you are working on only one side of an equation. If you are working on only one side of an equation (usually to simplify some expression), you would use the equal sign to connect the related steps.

EXAMPLE 2. Simplify the right-hand side of the equation $2y = 4x - x^2p + 1$.

There are like terms with respect to x on the right-hand side, so we can simplify by combining these terms.

$$2y = 4x - x^2p + 1$$

We connect the manipulations using equal signs because we are manipulating only one side of the equation in each step..

$$= 4x - x \cdot (xp) + 1$$ ***(Rewrite x^2p so that it has x as a factor.)***

$$= (4 - xp)x + 1$$ ***(Combine like terms.)***

In the exercises below, solve the equation for the indicated expression. Follow the procedure in Example 1. That is,

- Equation one arithmetic step at a time, and place each step on a separate line.
- Use implication arrows correctly.
- Note your arithmetic or simplification step to the right of each equation.

Exercise 1 Solve the equation $3x - 2y = x + 1$ for the variable x.

Exercise 2 Solve the equation $\sqrt{2}\,x + 2tp = x - 1$ for the variable x.

Exercise 3 Solve the equation $\sqrt{2}\,x + 2tp = x - 1$ for the expression tp .

Exercise 4 Consider the equation $2t + 7 = 3y - 5yt$.

Part (a) First, simplify the right-hand side of the equation by combining like terms with respect to y using equal signs correctly as in Example 2.

Part (b) Solve the simplified equation for the variable y.

Exercise 5

Suleiman wants to simplify the right-hand side of the equation

$$w - 5 = 2t + 7t^2 - 6$$

Part (a)

Explain what is incorrect about Suleiman's procedure below.

$$w - 5 = 2t + 7t^2 - 6 \implies 2t + t \cdot (7t) - 6 \implies (2 + 7t)t - 6$$

Part (b)

Write the simplification correctly.

Exercise 6

Reisha wants to solve the equation $w - 5 = 2t + 7t^2 - 6$ for w.

Part (a)

Explain what is incorrect about Reisha's procedure below.

$$w - 5 = 2t + 7t^2 - 6 = w = 2t + 7t^2 - 1$$

Part (b)

Solve the equation correctly.

An equation is ***linear*** with respect to a particular expression provided the following conditions are met:

- No term in the equation is divided by the expression.
- The expression is never raised to a power other than 1 in the equation.
- No root of the expression is taken.

EXAMPLE 3. Consider the equation $3xt - \frac{5}{x} + 2\sqrt{t-1} = 5p$.

- This equation is not linear with respect to x, because the middle term on the left-hand side is divided by x.
- This equation is not linear with respect to t, because t is part of a square root.
- This equation is linear with respect to p.
- This equation is linear with respect to xt.

Exercise 7 Consider the equation $u^3 - 3xu = 5 - \frac{x}{v}$.

Part (a) Is this equation linear with respect to x?

Part (b) Is this equation linear with respect to u?

Part (b) Is this equation linear with respect to v?

Exercise 8 Consider the equation $(t-1)pry = r^4(t-1) + \frac{3y}{t-1}$.

Part (a) Is this equation linear with respect to $t - 1$?

Part (b) Is this equation linear with respect to r^4?

Part (b) Is this equation linear with respect to y?

The equations in Exercises 1 – 6 are all linear with respect to the expression you are asked to solve for.

Exercise 9 Consider the equation $4p^2 - 3t = 7t + 3p$.

Part (a) Is this equation linear with respect to t ?

Part (b) Is this equation linear with respect to p ?

Part (c) Solve the equation for the variable that it is linear with respect to. Follow the same careful procedure you used in Problems 1 – 6.

There is a twist we have to keep in mind when solving any equation for a given expression. In the process of solving for that expression, we may have to divide both sides of the Equation by some other expression *that can vary*.

Division by the value 0 is never allowed in algebra.

If the process of solving an equation for some expression results in division by another expression that can vary, we have to exclude any values of the variables that cause the expression we divided by to equal 0.

EXAMPLE 4. In Exercise 4, you were asked to solve the equation $2t + 7 = 3y - 5yt$ for y . When you did, you found that the solution is

$$y = \frac{2t + 7}{3 - 5t}$$

It is possible for the expression $3 - 5t$ to equal 0. We must exclude any values of t that can cause this to happen.

$$3 - 5t = 0 \implies (3 - 5t) - 3 = 0 - 3$$ *(Subtract 3 from both sides)*

$$\implies (3 - 3) - 5t = -3$$ *(Combine like terms.)*

$$\implies 0 - 5t = -3$$ *(Simplify.)*

$$\implies \frac{-5t}{-5} = \frac{-3}{-5}$$ *(Divide both sides by -5.)*

$$\implies t = \frac{3}{5}$$ *(Simplify.)*

EXAMPLE 4 Continued

Since $t = {}^{3}/_{5}$ causes the expression $3 - 5t$ to equal 0, we must exclude this value of t from the solution.

Therefore, the **actual** solution to the equation $2t + 7 = 3y - 5yt$ will be

$$y = \frac{2t+7}{3-5t} \quad \text{as long as } t \neq \frac{3}{5}$$

Exercise 10 Consider the equation $2p - 7xp = x - t$.

Part (a) Solve this equation for p and be careful to exclude values of any variables that cause division by 0.

Part (b) Solve this equation for x and be careful to exclude values of any variables that cause division by 0.

Exercise 11 Consider the equation $uv - 8u = 2u - 5pv + 6$. Solve this equation for u And be careful to exclude values of any variables that cause division by 0.