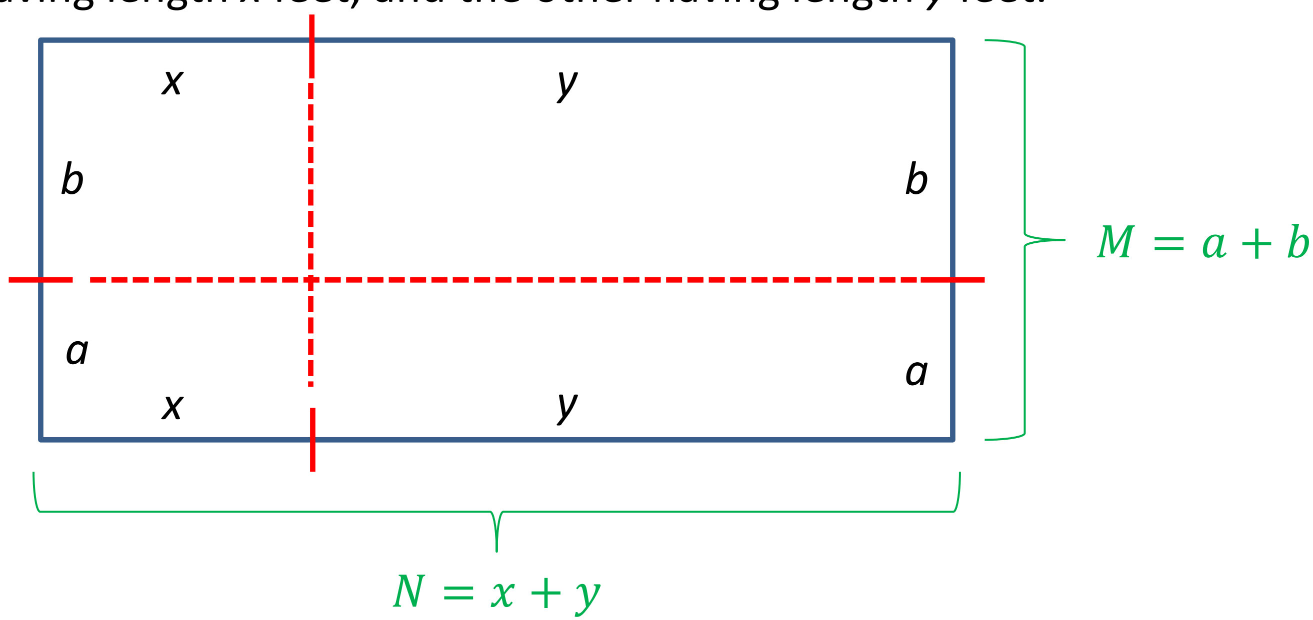
ALGEBRA REVIEW
UNIT 3 --- The Distributive Property

In this unit, we will explore one of the most important tools used in algebra. This time, we begin with an example.

Suppose one side of a rectangle has length *M* feet, and one side has length *N* feet. We know the area of this rectangle is represented by the product *MN*.

Now, suppose we divide the side of length *M* feet into two pieces, one having length *a* feet, and the other having length *b* feet. Suppose we also divide the side of length *N* feet into two pieces, one having length *x* feet, and the other having length *y* feet.



Exercise 1 Explain why the rectangle diagram on the previous page tells us

$$(a+b)(x+y) = MN = ax + ay + bx + by$$

The rectangle example motivates the following property relating multiplication of real numbers and addition of real numbers.

The Distributive Property

If a, b, x, y are any real numbers, then the equation

$$(a+b)(x+y) = ax + ay + bx + by$$

will always be valid.

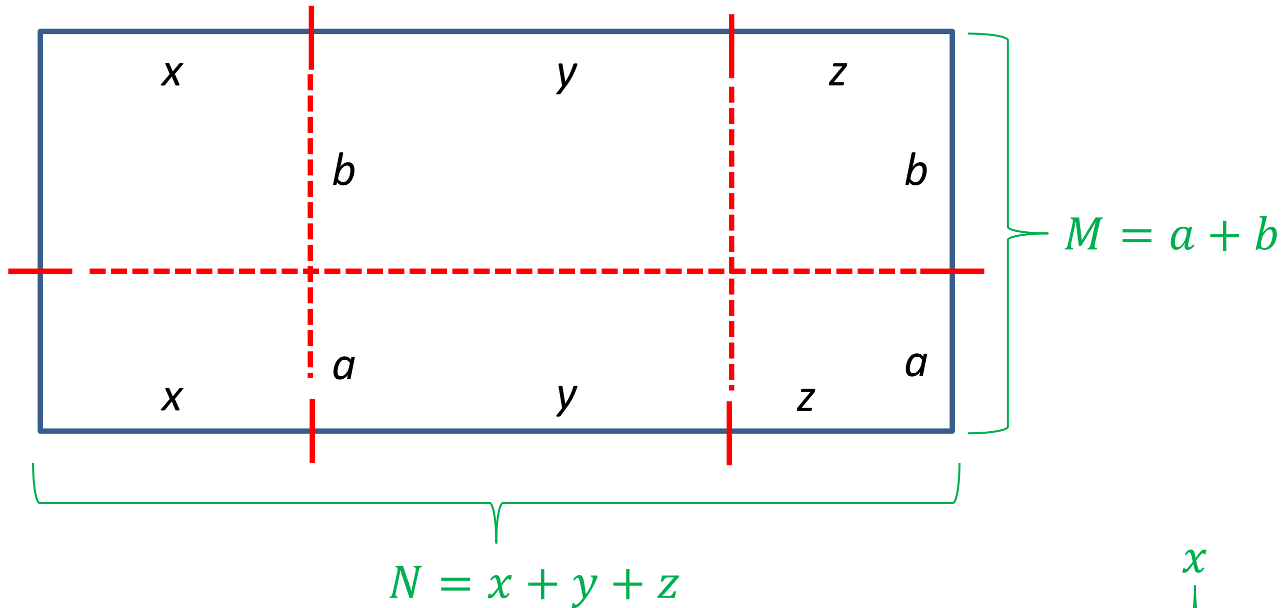
Applying the distributive property to a product is called ***expanding*** the product.

Exercise 2 Expand the product $(2+u)(3+u)$. Combine any like terms in your final answer.

Exercise 3 Expand the product $(u-t)(4+y)$. Combine any like terms in your final answer.

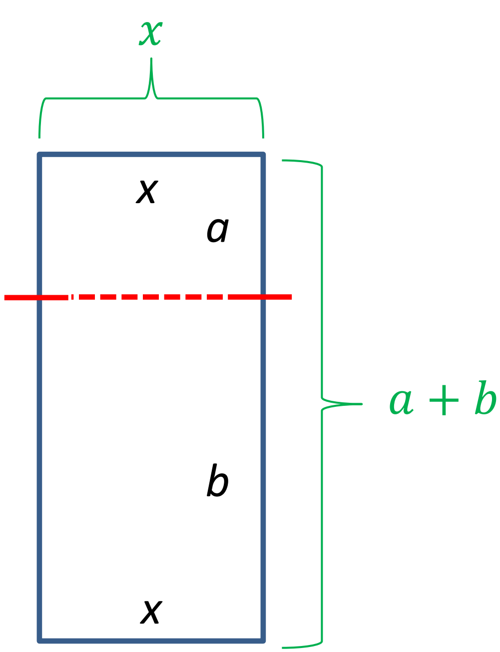
Exercise 4

Use the rectangle diagram below to determine what expanding the product $(a+b)(x+y+z)$ should give us.



Exercise 5

Use the rectangle diagram on the right to determine what expanding the product $(a+b)x$ should give us.



When we multiply two sums together, we multiply every term in one sum by every term in the other sum and add all of these products together.

EXAMPLE 1. Expand the product $(2-y)(3+x-t)$.

$$(2-y)(3+x-t) = 2\cdot 3 + \ \ldots$$

$$(2-y)(3+x-t) = 2\cdot 3 + 2\cdot x \ + \ \ldots$$

$$(2-y)(3+x-t) = 2\cdot 3 + 2\cdot x \ + 2\cdot(-t) + \ \ldots$$

$$(2-y)(3+x-t) = 2\cdot 3 + 2\cdot x \ + 2\cdot(-t) + (-y)\cdot 3 + \ \ldots$$

$$(2-y)(3+x-t) = 2\cdot 3 + 2\cdot x \ + 2\cdot(-t) + (-y)\cdot 3 + (-y)\cdot x + \ldots$$

$$(2-y)(3+x-t) = 2\cdot 3 + 2\cdot x \ + 2\cdot(-t) + (-y)\cdot 3 + (-y)\cdot x + (-y)\cdot(-t)$$

EXAMPLE 1 Continued

Once we have multiplied every term in the left-hand sum by every term in the right-hand sum and added all of the resulting products together, we simplify to obtain the expansion.

$$(2-y)(3+x-t) = 6+2x-2t-3y-yx+yt$$

Exercise 6

Expand the product $2t(3-y-t)$. Simplify your expansion as much as possible.

Exercise 7

Expand the product $(3w+5)(x+w-1)$. Simplify your expansion as much as possible.

Exercise 8

Trevor believes that $(a+b)^2 = a^2+b^2$. Explain why this is incorrect. What is the correct equation?

Exercise 9

Nancy wants to simplify the expression $2x-3(x+4)$. After some work, she comes up with the equation $2x-3(x+4) = 4-x$. What mistakes did Nancy make? What is the correct equation?

Exercise 10 Expand the product $(x-y)(x^2+xy+y^2)$. Simplify your expansion as much as possible.

Exercise 11 Solve the equation $2x-3(x+4)=5$ for x.

Exercise 12 Solve the equation $4(3-w)-2(w-1)=w$ for w.

Exercise 13 Solve the equation $(t-1)(2z+3)=z+1$ for z. Exclude any values of t that cause division by 0 in your solution.

Exercise 14 Solve the equation $(t-1)(2z+3)=z+1$ for t. Exclude any values of z that cause division by 0 in your solution.

Exercise 15 Solve the equation $m(3x+1)-x(m-4)=0$ for m. Exclude any values of x that cause division by 0 in your solution.

Exercise 16 Solve the equation $m(3x+1)-x(m-4)=0$ for x. Exclude any values of m that cause division by 0 in your solution.