BIG SHEET OF DERIVATIVE PRACTICE

Here are some basic facts to remember.

- A function $y = f(x)$ is *differentiable* at an input value $x = a$ provided

  1. The function is defined at $x = a$; that is $(a, f(a))$ is a point on the graph of f.
  2. When you zoom in repeatedly on the graph of f, focused on the point $(a, f(a))$, the graph of f looks more and more like a straight line.

  This straight line is called the *tangent line* to the graph of f at the point $(a, f(a))$.

- The slope of the tangent line to the graph of a function $y = f(x)$ at the point $(a, f(a))$ is called the *derivative* of the function f at $x = a$. The derivative of f at $x = a$ is denoted by the symbol $f'(a)$. The process of determining $f'(a)$ is called *differentiation*.

- The formula for the tangent line to the graph of $y = f(x)$ at the point $(a, f(a))$ is always given by $y - f(a) = f'(a)\,[x - a]$.

- The derivative of a function f at the at the input value $x = a$ can always be found by evaluating the limiting process

$$\lim_{x \longrightarrow a} \frac{f(x) - f(a)}{x - a} \qquad \text{OR} \qquad \lim_{h \longrightarrow 0} \frac{f(a+h) - f(a)}{h}$$

- The *derivative function* for a function $y = f(x)$ is a new function whose output is slope of the tangent line to f for any input value x. The derivative function for f is denoted by

$$f' \qquad \text{OR} \qquad \frac{df}{dx}$$

- If a function f has a local maximum or minimum output at $x = a$, then $f'(a)$ will either be undefined or will be equal to 0.

- If a function f has an inflection point at $x = b$, then $f'(a)$ will be a local maximum or minimum output for the derivative function f'.

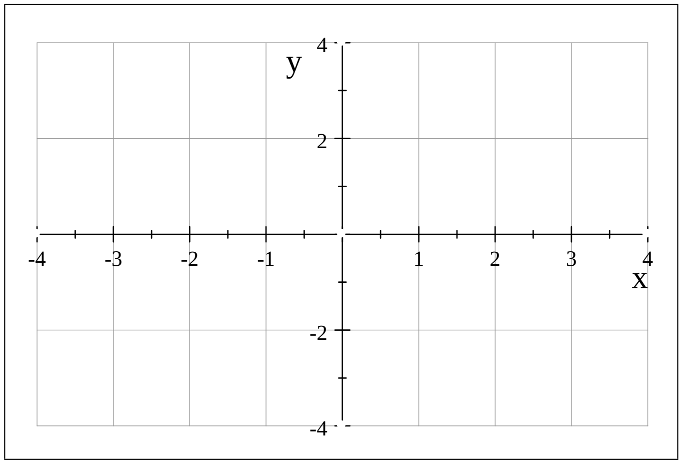
- If the graph of f is increasing on an interval, then the graph of f' will be above the horizontal axis on this interval.

- If the graph of f is decreasing on an interval, then the graph of f' will be below the horizontal axis on this interval.

EXTRA PROBLEMS FOR SECTION 2.8

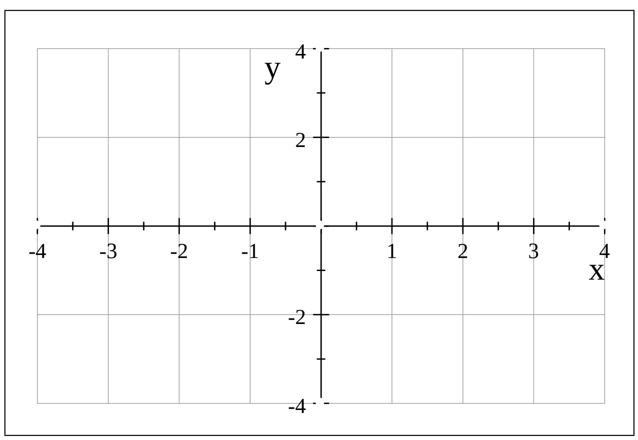
1. On the grid provided, sketch the graph of a function $y = f(x)$ that satisfies the following conditions. (There are many possible answers.)

   - The tangent line to the graph of f is horizontal at the input values $x = -1$ and $x = 2$.
   - The graph of the derivative function f' is above the x-axis on the interval $-1 < x < 2$ and is below the x-axis for all other input values except for $x = -1$ and $x = 2$.



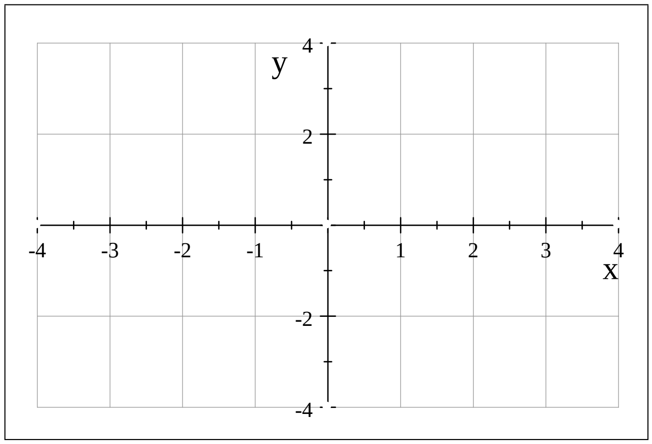
2. On the grid provided, sketch the graph of a function $y = f(x)$ that satisfies the following conditions. (There are many possible answers.)

   - The tangent line to the graph of f is horizontal at the input value $x = -3$ and is undefined at the input value $x = 3$.
   - The graph of the derivative function f' is below the x-axis on the interval $-\infty < x < -3$ and on the interval $3 < x < +\infty$, and is above the x-axis on the interval $-3 < x < 3$.

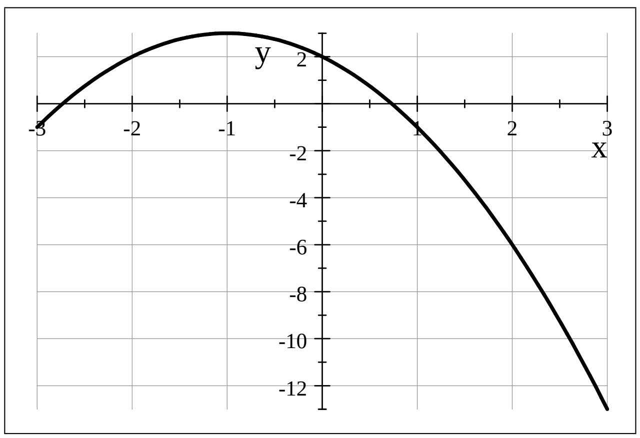


3. On the grid provided, sketch the graph of a function $y = f(x)$ that satisfies the following conditions. (There are many possible answers.)

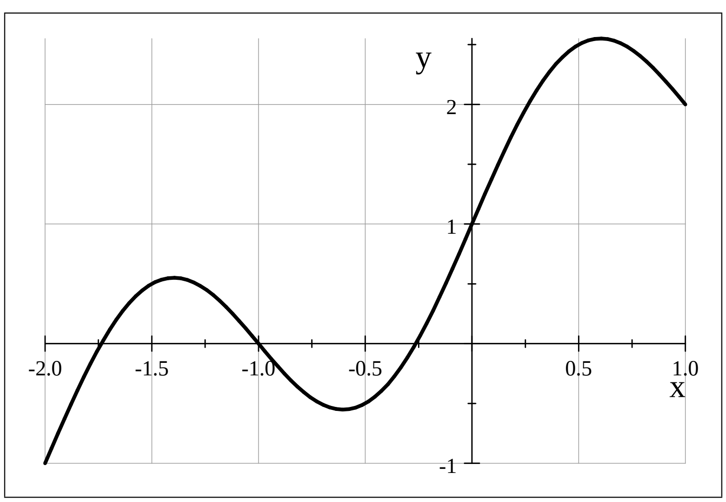
   - The tangent line to the graph of f is horizontal at the input value $x = 0$.
   - The graph of the derivative function f' is below the x-axis for all input values except $x = 0$.



4. The diagram below shows the graph of the derivative function for a function $y = f(x)$. At which input values does the function f have its local maximum output values?



5. The diagram below shows the graph of the derivative function for a function $y = f(x)$. At which input values does the function f have its local minimum output values?



6. Consider the function $y = f(x)$ whose derivative graph is shown in Problem 5. Suppose we know that $f(-1.5) = 3.6$

   (a) What is the formula for the line tangent to the graph of the function f at the point $(-1.5, 3.6)$?

   (b) What is the approximate value of $f(-1.2)$?

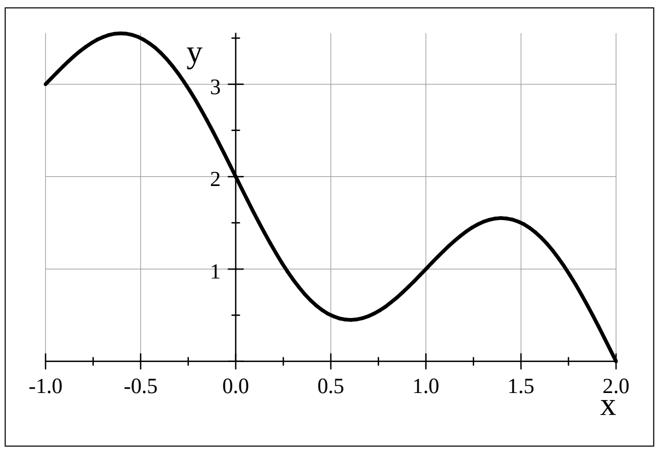
7. Consider the function $y = f(x) = \cos(\cos(\cos(x)))$. Use the limit definition of the derivative function for f to estimate the value of $f'(2)$ accurate to at least two decimal places.

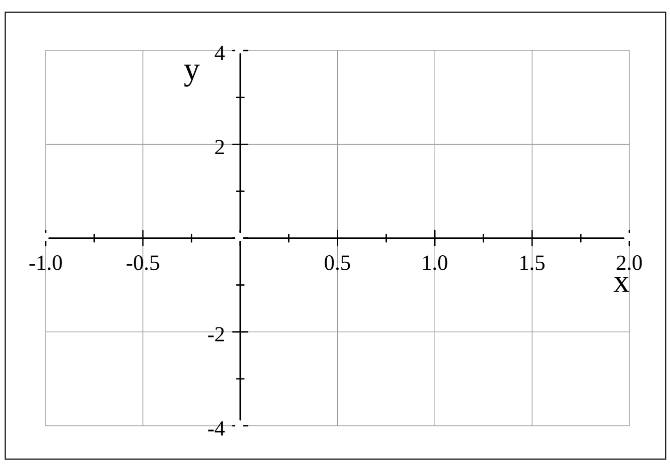
8. Consider the function $y = f(x) = \sin(e^{\cos(x)})$. Use the limit definition of the derivative function for f to estimate the value of $f'(1)$ accurate to at least three decimal places.

9. Use the limit definition of the derivative to determine the formula for the derivative function of the function $y = f(x)$ defined by

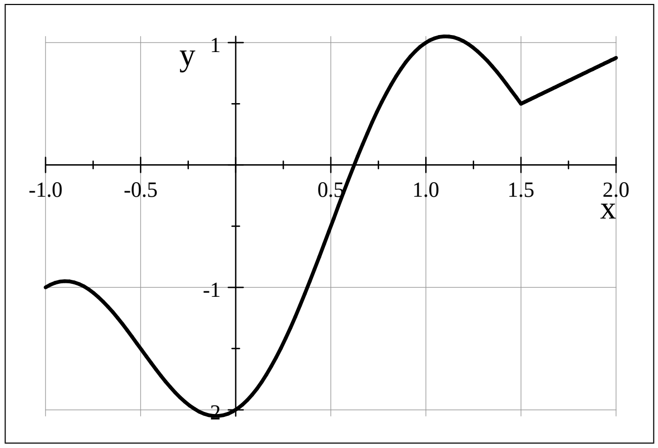
$$f(x) = \frac{1}{(x-1)^2}$$

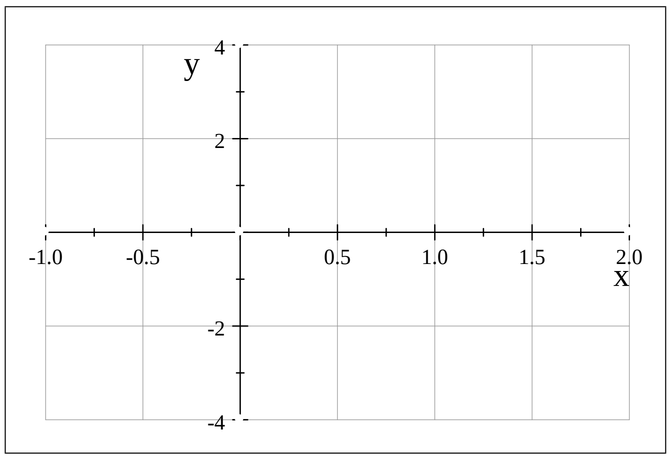
10. The diagram below shows the graph of a function $y = f(x)$. On the grid provided, draw a rough sketch of the graph of the derivative function for the function f.





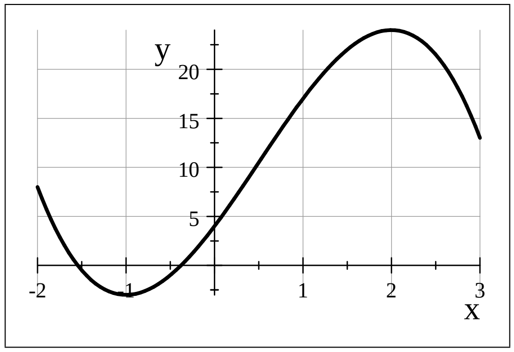
11. The diagram below shows the graph of a function $y = f(x)$. On the grid provided, draw a rough sketch of the graph of the derivative function for the function f.





ANSWERS

1. Based on the information given, the function f will have a local minimum output at $x = -1$ and a local maximum output at $x = 2$. There are no other turning points for the function f, and there are no kinks, holes, or vertical asymptotes. Any graph that satisfies these criteria would suffice. (The graph of the function $y = f(x) = 4 - 2x^3 + 3x^2 + 12x$ is one example.)

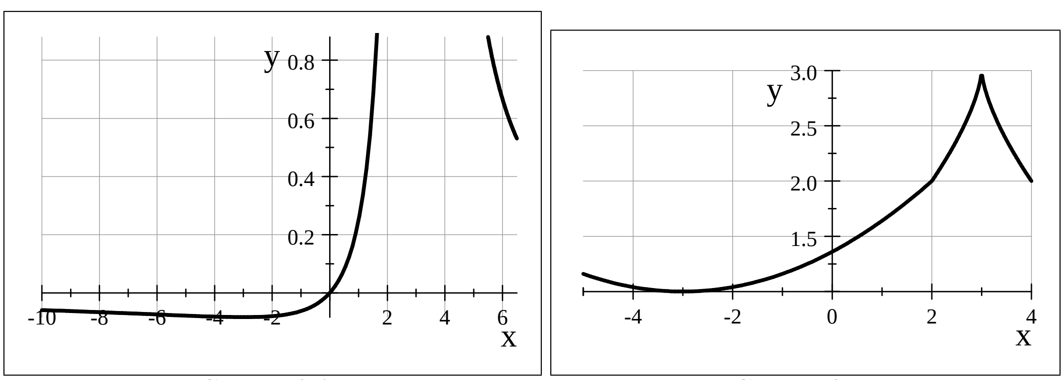


2. Based on the information given, the function f will have a local minimum output at $x = -3$. The graph of the function f should be increasing on the input interval $0 < x < 3$ and should be decreasing on the input interval $3 < x < +\infty$. The function f *could* have a vertical asymptote at $x = 3$, provided

$$\lim_{x \to 3^-} f(x) = +\infty = \lim_{x \to 3^+} f(x)$$

(The graph of the function $y = f(x) = \dfrac{x}{(x-3)^2}$ would be one example.) It is not necessary for the graph of f to have a vertical asymptote, however. It is also possible for the graph of the function to have a kink at $x = 3$; in this case, the function must have a local maximum output at $x = 3$. One example would be the graph of the function

$$y = f(x) = \begin{cases} 1 + \dfrac{(x+3)^2}{25} & \text{if} \quad x \le 2 \\ 3 - (x-3)^{2/3} & \text{if} \quad x > 2 \end{cases}$$



Graph of f Graph of g

3. Based on the information given, the function f has no turning points, and the graph of the function must be decreasing. However, the graph of the function must have a horizontal tangent line at the point $(0, f(0))$. The easiest way to accomplish this is to let the function f have an inflection point at $(0, f(0))$. The function $y = f(x) = -x^3$ is an example of one such function.

4. Since $f'(x) = 0$ when $x \approx -2.75$ and $x \approx 0.75$, it is at these input values where the function f might have turning points. The graph of the derivative function tells us that the graph of the function f is increasing on the input interval $-2.75 < x < 0.75$ and is decreasing on the input interval $0.75 < x < +\infty$. Hence, the function f will have a local maximum output when $x \approx 0.75$.

5. Using the same method of analysis as in Problem 4, the function f will have local minimum output when $x \approx -1.74$ and $x \approx -0.26$.

6. According to the graph of the derivative function, we see that $f'(-1.5) \approx 0.5$. Therefore, the formula for the tangent line would be $y \approx 0.5(x + 1.5) + 3.6$. Since the graph of the tangent line is a good approximation to the graph of the function f near the point of tangency, we know that $f(-1.2) \approx 0.5(-1.2 + 1.5) + 3.6 = 3.75$.

7. Consider the average rate of change function

$$g_2(h) = \frac{\cos(\cos(\cos(2+h))) - \cos(\cos(\cos(2)))}{h}$$

Using the table feature on the graphing calculator, we find that $g_2(0.001) \approx 0.2914$ is accurate to at least two decimal places.

8. Consider the average rate of change function

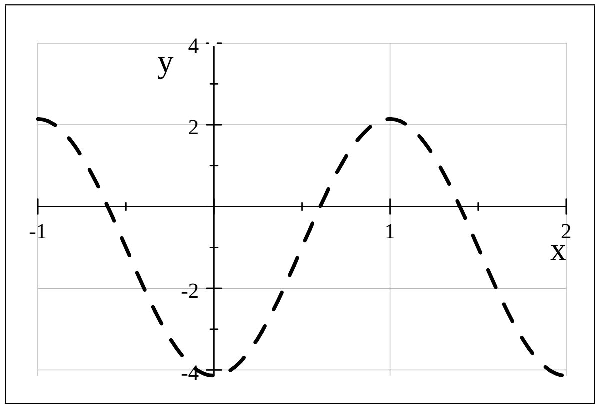
$$g_1(h) = \frac{\sin\left(e^{\cos(1+h)}\right) - \sin(e^{\cos(1)})}{h}$$

Using the table feature on the graphing calculator, we find that $g_2(0.000001) \approx 0.2097$ is accurate to at least three decimal places.

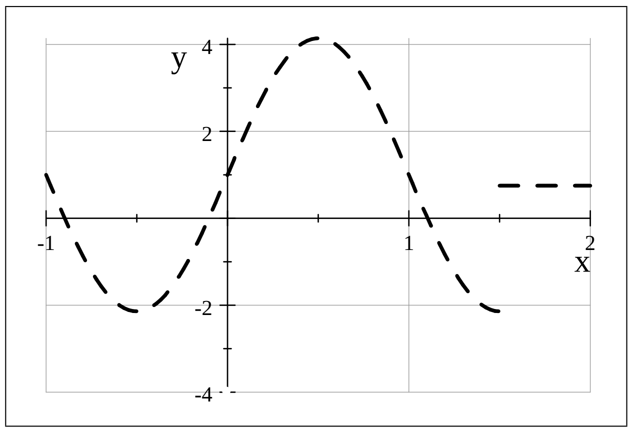
9. Observe that

$$\begin{aligned}
f'(x) &= \lim_{h\to 0}\left(\frac{1}{h}\right)\left[\frac{1}{(x+h-1)^2} - \frac{1}{(x-1)^2}\right] \\
&= \lim_{h\to 0}\left(\frac{1}{h}\right)\left[\frac{(x-1)^2 - (x+h-1)^2}{(x+h-1)^2(x-1)^2}\right] \\
&= \lim_{h\to 0}\left(\frac{1}{h}\right)\left[\frac{x^2 - 2x + 1 - (x+h)^2 + 2(x+h) - 1}{(x+h-1)^2(x-1)^2}\right] \\
&= \lim_{h\to 0}\left(\frac{1}{h}\right)\left[\frac{x^2 - 2x + 1 - x^2 - 2xh - h^2 + 2x + 2h - 1}{(x+h-1)^2(x-1)^2}\right] \\
&= \lim_{h\to 0}\left(\frac{1}{h}\right)\left[\frac{-2xh - h^2 + 2h}{(x+h-1)^2(x-1)^2}\right] \\
&= \lim_{h\to 0}\left(\frac{h}{h}\right)\left[\frac{-2x - h + 2}{(x+h-1)^2(x-1)^2}\right] \\
&= \lim_{h\to 0}\left[\frac{-2x - h + 2}{(x+h-1)^2(x-1)^2}\right] \\
&= -\frac{2(x-1)}{(x-1)^4} \\
&= -\frac{2}{(x-1)^3}
\end{aligned}$$

10. The derivative graph would look like



11. The derivative graph would look like



SHORTCUTS TO DIFFERENTIATION

Using the limit definition to differentiate a function f is a very tedious process. Fortunately, this limiting process displays consistent properties that allow us to take shortcuts. These shortcuts fall into two classes.

SPECIAL FORMULAS: These shortcuts tell us how to differentiate functions that are given by specific formulas.

GENERAL RULES: These shortcuts tell us how to differentiate combinations of functions.

When confronted with a function f we want to differentiate, we always follow the same stratetgy.

1. Use the General Rules to break up the derivative process for f into simpler and simpler derivative processes until we are faced only with specific formulas.
2. Apply the Special Rules to differentiate the specific formulas.

SPECIFIC DERIVATIVE FORMULAS (Sections 3.1 - 3.3)

1. The derivative function for any constant function is the zero-function. That is, $\dfrac{d}{dx}[K] = 0$ for any fixed real number K.
2. If n is any rational number, then $\dfrac{d}{dx}[x^n] = nx^{n-1}$.
3. If a is any positive real number, then $\dfrac{d}{dx}[a^x] = a^x \cdot \ln(a)$.
4. We have $\dfrac{d}{dx}[\sin(x)] = \cos(x)$, $\dfrac{d}{dx}[\cos(x)] = -\sin(x)$, and $\dfrac{d}{dx}[\tan(x)] = \sec^2(x)$.

GENERAL RULES (Sections 3.1 - 3.3)

Constant Multiple Rule: If $y = f\ (x)$ is a differentiable function and K is any fixed real number, then

$$\underbrace{(K \cdot f)' = K \cdot f'}_{\text{Prime Notation}} \qquad \text{OR} \qquad \underbrace{\frac{d}{dx}[K \cdot f(x)] = K \cdot \frac{d}{dx}[f(x)]}_{\text{Differential Notation}}$$

Sum Rule: If $y = f(x)$ and $y = g(x)$ are any differentiable functions, then the derivative function for $f+g$ is the sum of the derivative functions for f and g. In symbols,

$$(f+g)' = f' + g' \qquad \text{OR} \qquad \frac{d}{dx}[f(x) + g(x)] = \frac{d}{dx}[f(x)] + \frac{d}{dx}[g(x)]$$

Product Rule: If $y = g(x)$ and $y = h(x)$ are differentiable functions, then their product $f = g \cdot h$ is also differentiable, and

$$\underbrace{(g \cdot h)' = g \cdot h' + h \cdot g'}_{\text{Prime Notation}} \qquad \text{OR} \qquad \underbrace{\frac{d}{dx}\left[g(x) \cdot h(x)\right] = g(x) \cdot \frac{d}{dx}\left[h(x)\right] + h(x) \cdot \frac{d}{dx}\left[g(x)\right]}_{\text{Differential Notation}}$$

Quotient Rule: If $y = g(x)$ and $y = h(x)$ are differentiable functions, then their quotient $f = \dfrac{g}{h}$ is also differentiable as long as f is defined, and

$$\underbrace{\left(\frac{g}{h}\right)' = \frac{h \cdot g' - g \cdot h'}{h^2}}_{\text{Prime Notation}} \qquad \text{OR} \qquad \underbrace{\frac{d}{dx}\left[\frac{g(x)}{h(x)}\right] = \left(\frac{1}{\left[h(x)\right]^2}\right)\left(h(x) \cdot \frac{d}{dx}\left[g(x)\right] - g(x) \cdot \frac{d}{dx}\left[h(x)\right]\right)}_{\text{Differential Notation}}$$

Example 1 *Differentiate the function* $g(a) = 2a^{-3} + 5^a \sin(a) + 9$.

Solution. The formula for g is constructed from sums and products of functions whose derivatives we know. Therefore, we will use the General Rules to break up the derivative process for g into simpler and simpler derivative processes, until we are only left with the derivatives of specific formulas.

$$\begin{aligned}
g'(a) &= \frac{d}{da}\left[2a^{-3} + 5^a \sin(a) + 9\right] && \\
&= \frac{d}{da}\left[2a^{-3}\right] + \frac{d}{da}\left[5^a \sin(a)\right] + \frac{d}{da}\left[9\right] && \text{Apply Sum Rule} \\
&= 2\frac{d}{da}\left[a^{-3}\right] + \frac{d}{da}\left[5^a \sin(a)\right] + \frac{d}{da}\left[9\right] && \text{Apply Constant Multiple Rule} \\
&= 2\frac{d}{da}\left[a^{-3}\right] + \left(5^a \cdot \frac{d}{da}\left[\sin(a)\right] + \sin(a) \cdot \frac{d}{da}\left[5^a\right]\right) + \frac{d}{da}\left[9\right] && \text{Apply Product Rule} \\
&= 2 \cdot \left(-3a^{-4}\right) + 5^a \cdot \cos(a) + \sin(a) \cdot 5^a \ln(5) + 0 && \text{Apply Specific Formulas} \\
&= -\frac{6}{a^4} + 5^a\left[\cos(a) + \ln(5)\sin(a)\right] &&
\end{aligned}$$

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EXTRA PROBLEMS FOR SECTIONS 3.1 - 3.3

Differentiate the following functions using the General Rules and the Specific Formulas.

(1) $f(x) = 3$	(2) $g(y) = 4y^2 - 1$	(3) $h(c) = 3c\sin(c)$
(4) $f(a) = 5a^{-4} + 2\cos(a)$	(5) $g(z) = \sin(z)\cos(z)$	(6) $h(u) = 4e^u - u \cdot 3^u$
(7) $f(t) = t\cos(t) + 3$	(8) $g(w) = w^2\tan(w)$	(9) $h(p) = \pi - 2p\tan(p)$
(10) $f(r) = e - 3r^{-1} + 4r^3 \cdot 5^r$	(11) $g(s) = \left(4s^2 - 1\right)^2$	(12) $h(v) = \left(v^3 + 5v\right)^2$
(13) $f(b) = \left(2b^2 - b + 1\right)^{-1}$	(14) $g(k) = \dfrac{k^2}{k-1}$	(15) $h(n) = \dfrac{\sin(n)}{3n+5}$

16. What is the formula for the tangent line to $h(u) = u\cos(u)$ at the point $(0, h(0))$?

17. For what values of y will the tangent line to the graph of $g(y) = \dfrac{y^2}{1+y}$ have slope -1?

18. For what values of x will the tangent line to the graph of $f(x) = (4x-8)^2$ be horizontal?

ANSWERS:

1. Apply Constant Formula — $f'(x) = 0$
2. Apply Sum Rule followed by specific formulas — $g'(y) = 8y$
3. Apply Constant Multiple Rule followed by Product Rule followed by specific formulas — $h'(c) = 3\sin(c) + 3c\cos(c)$
4. Apply Sum Rule followed by Constant Multiple Rule followed by specific formulas — $f'(a) = -20a^{-5} - 2\sin(a)$
5. Apply Product Rule followed by specific formulas — $g'(z) = \cos^2(z) - \sin^2(z)$
6. Apply Sum Rule followed by Constant Multiple Rule followed by Product Rule followed by specific formulas — $h'(u) = 4e^u - 3^u(1 + u\ln(3))$
7. Apply Sum Rule followed by Product Rule followed by specific formulas — $f'(t) = \cos(t) - t\sin(t)$
8. Apply Product Rule followed by specific formulas — $g'(w) = 2w\tan(w) + w^2\sec^2(w)$
9. Apply Sum Rule followed by Constant Multiple Rule followed by Product Rule followed by specific formulas — $h'(p) = -2\tan(p) - 2p\sec^2(p)$
10. Apply Sum Rule followed by Constant Multiple Rule followed by Product Rule followed by specific formulas — $f'(r) = 3r^{-2} + 5^r\left(12r^2 + 4r^3\ln(5)\right)$
11. Apply Product Rule followed by Sum Rule followed by Constant Multiple Rule followed by specific formulas — $g'(s) = 16s(4s^2 - 1)$
12. Apply Product Rule followed by Sum Rule followed by Constant Multiple Rule followed by specific formulas — $h'(v) = (6v^2 + 10)(v^3 + 5v)$
13. Apply Quotient Rule followed by Sum Rule followed by Constant Multiple Rule followed by specific formulas — $f'(b) = -\dfrac{4b-1}{(2b^2 - b + 1)^2}$
14. Apply Quotient Rule followed by Sum Rule followed by specific formulas — $g'(k) = \dfrac{k(k-2)}{(k-1)^2}$
15. Apply Quotient Rule followed by Sum Rule followed by Constant Multiple Rule followed by specific formulas — $h'(n) = \dfrac{(3n+5)\cos(n) - 3\sin(n)}{(3n+5)^2}$
16. The point of tangency will be $(0, 0)$. Now, $h'(u) = \cos(u) - u\sin(u)$. Therefore, the slope of the tangent line will be $h'(0) = 1$. The equation of the tangent line will be $y = u$.
17. First, note that

$$g'(y) = \frac{y^2 + 2y}{(1+y)^2}$$

Now, we want $g'(y) = -1$. Observe

$$\begin{aligned} -1 = \frac{y^2 + 2y}{(1+y)^2} &\implies -(1 + 2y + y^2) = y^2 + 2y \\ &\implies 0 = 1 + 4y + 2y^2 \\ &\implies y = -1 \pm \frac{\sqrt{2}}{2} \qquad \text{(Apply Quadratic Formula)} \end{aligned}$$

18. First note that $f'(x) = 32x - 64$. We want $f'(x) = 0$. Observe

$$32x - 64 = 0 \implies x = 2$$

The last of the general rules for differentiation is the *Chain Rule*. The Chain Rule is a shortcut to the limit definition of the differentiation process that tells us how to find the derivative of composite functions.

CHAIN RULE: If $y = g(x)$ and $v = h(u)$ are differentiable functions, then the function $f = h \circ g$ is a differentiable, and

$$\frac{df}{dx} = \frac{dg}{dx} \cdot \left.\frac{dh}{du}\right|_{u=g(x)}$$

Example 2 *Differentiate the function* $g(x) = \cos^2(x^3)$.

Solution. The function g is actually a composition of three functions, since

$$f(x) = \big[\cos(x^3)\big]^2$$

We assign a function name to each formula in the composition. Working from the inside out, let

- $v(x) = x^3$ so that $\dfrac{dv}{dx} = 3x^2$
- $u(w) = \cos(v)$ so that $\dfrac{du}{dv} = -\sin(v)$
- $f(u) = v^2$ so that $\dfrac{df}{du} = 2u$

Observe that $g(x) = f(u(v(x))$. Now, according to the Chain Rule, we have

$$\begin{aligned} \frac{d}{dx}\big[\cos^2(x^3)\big] &= \left.\frac{df}{du}\right|_{u=\cos(x^3)} \cdot \left.\frac{du}{dv}\right|_{v=x^3} \cdot \frac{dv}{dx} \\ &= \big[2\cos(x^3)\big] \cdot \big[-\sin(x^3)\big] \cdot \big[3x^2\big] \\ &= -6x^2\cos(x^3)\sin(x^3) \end{aligned}$$

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EXTRA PROBLEMS FOR SECTION 3.4

Use the Chain Rule to differentiate the following functions.

(1) $f(x) = \sin(4x+1)$	(2) $g(x) = \tan^2(x)$	(3) $h(x) = \big(x^2-1\big)^{-4}$
(4) $f(x) = \cos^4(x)$	(5) $g(x) = (5x-4)^{3/4}$	(6) $h(x) = \tan(e^x)$
(7) $f(x) = \cos(x+2^x)$	(8) $g(x) = 3^{\sin(x)}$	(9) $h(x) = x\cos^2(x)$
(10) $f(x) = 4^x\tan(3x)$	(11) $g(x) = \tan^3(x^3)$	(12) $h(x) = (x-\sin(3x))^{-2}$

13. What is the instantaneous rate of change for the function $y = f(x) = (x^2-1)^{4/3}$ at $x = 3$?

14. What is the formula for the line tangent to the function $u = g(v) = v\cos^3(v)$ at the point $(1, g(1))$?

15. Are there any input values where the instantaneous rate of change for the function $w = h(r) = (3r-1)^2$ is equal to 0?

16. Are there any input values where the slope of the tangent line to the graph of the function $m = g(n) = (3n^2-1)^{2/3}$ is undefined?

ANSWERS

1. Let $u(x) = 4x+1$. We have $f'(x) = \frac{d}{du}[\sin(u)] \cdot \frac{d}{dx}[4x+1] = 4\cos(4x+1)$.

2. Let $u(x) = \tan(x)$. We have $g'(x) = \frac{d}{du}\left[u^2\right] \cdot \frac{d}{dx}[\tan(x)] = 2\tan(x)\sec^2(x)$.

3. Let $u(x) = x^2-1$. We have $h'(x) = \frac{d}{du}\left[u^{-4}\right] \cdot \frac{d}{dx}\left[x^2-1\right] = -\frac{8}{(x^2-1)^5}$.

4. Let $u(x) = \cos(x)$. We have $f'(x) = \frac{d}{du}\left[u^4\right] \cdot \frac{d}{dx}[\cos(x)] = -4\cos^3(x)\sin(x)$.

5. Let $u(x) = 5x-4$. We have $g'(x) = \frac{d}{du}\left[u^{3/4}\right] \cdot \frac{d}{dx}[5x-4] = \frac{15}{4}(5x-4)^{-1/4}$.

6. Let $u(x) = e^x$ We have $h'(x) = \frac{d}{du}[\tan(u)] \cdot \frac{d}{dx}[e^x] = \sec^2(e^x) \cdot e^x$.

7. Let $u(x) = x+2^x$. We have $f'(x) = \frac{d}{du}[\cos(u)] \cdot \frac{d}{dx}[x+2^x] = -(1+2^x\ln(2))\sin(x+2^x)$.

8. Let $u(x) = \sin(x)$. We have $g'(x) = \frac{d}{du}[3^u] \cdot \frac{d}{dx}[\sin(x)] = 3^{\sin(x)}\ln(3) \cdot \cos(x)$.

9. Let $u(x) = \cos(x)$. We have
$$h'(x) = \frac{d}{dx}[x]\cos^2(x) + x \cdot \frac{d}{du}\left[u^2\right] \cdot \frac{d}{dx}[\cos(x)] = \cos^2(x) - 2x\cos(x)\sin(x).$$

10. Let $u = 3x$. We have
$$f'(x) = \frac{d}{dx}[4^x]\tan(3x) + 4^x \cdot \frac{d}{du}[\tan(u)] \cdot \frac{d}{dx}[3x] = 4^x\left[\tan(3x)\ln(4) + 3\sec^2(3x)\right]$$

11. Let $u = x^3$ and let $v = \tan(u)$. We have
$$g'(x) = \frac{d}{dx}\left[x^3\right] \cdot \frac{d}{du}[\tan(u)] \cdot \frac{d}{dv}\left[v^3\right] = 9x^2\sec^2(x^3)\tan^2(x^3)$$

12. It is best to solve this problem in two steps. First, let $u = 3x$ and observe
$$\frac{d}{dx}[x-\sin(3x)] = \frac{d}{dx}[x] - \frac{d}{du}[\sin(u)] \cdot \frac{d}{dx}[3x] = 1 - 3\cos(3x)$$
Now, let $v = x-\sin(3x)$ and observe
$$h'(x) = \frac{d}{dv}\left[v^{-2}\right] \cdot \frac{d}{dx}[x-\sin(3x)] = -2[x-\sin(3x)]^{-3}(1-3\cos(3x))$$

13. First, note that $f'(x) = \dfrac{8x}{3}\sqrt[3]{(x^2-1)}$. Therefore, $f'(3) = 16$.

14. First, note that $g'(v) = \cos^3(v) - 3v\cos^2(v)\sin(v)$. Therefore, the formula for the tangent line will be $u \approx -0.579(v-1) + 0.1577$.

15. First, note that $h'(r) = 6(3r-1) = 18r - 6$. Therefore, setting $h'(r) = 0$ tells us that $r = 1/3$.

16. First, note that

$$g'(n) = \frac{4n}{\sqrt[3]{3n^2 - 1}}$$

Therefore, we know that $g'(n)$ will be undefined when $3n^2 - 1 = 0$; that is, when $n = \pm\frac{1}{\sqrt{3}}$.