Problem 1._____C______ The function f below has a removable discontinuity at the input value $b = 0$. Which of the following functions produces the same output as the function f for all nonzero input values?

$$y = f(b) = \left(\frac{1}{b}\right)\left(\frac{1}{b^2 - 1} + \frac{1}{b^2 + 1}\right)$$

(a) $g(b) = \dfrac{1}{b - 1} + \dfrac{1}{b^2 + 1}$

(b) $h(b) = \dfrac{1}{b - 1} + \dfrac{1}{b + 1}$

(c) $k(b) = \dfrac{2b}{(b^2 - 1)(b^2 + 1)}$

(d) $j(b) = \dfrac{2}{b^2 - 1}$

(e) $m(b) = \dfrac{1}{b - 1}$

$$\begin{aligned}\left(\frac{1}{b}\right)\left(\frac{1}{b^2 - 1} + \frac{1}{b^2 + 1}\right) &= \left(\frac{1}{b}\right)\left(\frac{1}{b^2 - 1}\left[\frac{\boldsymbol{b^2 + 1}}{\boldsymbol{b^2 + 1}}\right] + \frac{1}{b^2 + 1}\left[\frac{\boldsymbol{b^2 - 1}}{\boldsymbol{b^2 - 1}}\right]\right) \\ &= \left(\frac{1}{b}\right)\left(\frac{b^2 + 1 + b^2 - 1}{(b^2 - 1)(b^2 + 1)}\right) \\ &= \left(\frac{1}{b}\right)\left(\frac{2b^2}{(b^2 - 1)(b^2 + 1)}\right) \\ &= \left(\frac{\boldsymbol{b}}{\boldsymbol{b}}\right)\left(\frac{2b}{(b^2 - 1)(b^2 + 1)}\right) \\ &= \frac{2b}{(b^2 - 1)(b^2 + 1)} \qquad (b \neq 0)\end{aligned}$$

Problem 2._____A______ What are the input values where the function below has a removable discontinuity?

$$y = f(t) = \frac{t^2 - 1}{t^2 + 2t + 1}$$

(a) There are no removable discontinuities

(b) Only at $t = 1$

(c) Only at $t = 1, t = 0,$ and $t = -1$

(d) Only at $t = 1$ and $t = -1$

(e) Only at $t = -1$

$$\frac{t^2 - 1}{t^2 + 2t + 1} = \frac{(t - 1)(\boldsymbol{t + 1})}{(t + 1)(\boldsymbol{t + 1})} = \frac{t - 1}{t + 1}$$

Note that the function f has a discontinuity only at the input value $t = -1$. The discontinuity cannot be removed by factoring and simplifying the formula; hence the discontinuity is a vertical asymptote.

Problem 3._____E______ What are the input values where the function below has a vertical asymptote?

$$y = f(t) = \frac{t^2 - 1}{t^2 + 2t + 1}$$

(a) There are no vertical asymptotes

(b) Only at $t = 1$

(c) Only at $t = 1, t = 0,$ and $t = -1$

(d) Only at $t = 1$ and $t = -1$

(e) Only at $t = -1$

Problem 4._____D______ Which function below provides the average rate of change for the function $y = f(x) = 3 - 2\sqrt[3]{t}$ on the input interval from $t = 27$ to $t = 27 + h$?

(a) $g_{27}(h) = \frac{3 + h}{h}$

(b) $g_{27}(h) = -\frac{6 + 2\sqrt[3]{27 + h}}{h}$

(c) $g_{27}(h) = 4$

(d) $g_{27}(h) = \frac{6 - 2\sqrt[3]{27 + h}}{h}$

(e) $g_{27}(h) = \frac{3 - 2\sqrt[3]{27 + h}}{3 - 2\sqrt[3]{27}}$

$$g_{27}(h) = \frac{f(27 + h) - f(27)}{h} = \frac{\left(3 - 2\sqrt[3]{27 + h}\right) - \left(3 - 2\sqrt[3]{27}\right)}{h} = \frac{6 - 2\sqrt[3]{27 + h}}{h}$$

Problems 5, 6, and 7 refer to the function whose formula is given below.

$$y = f(x) = \begin{cases} \frac{3}{x^2} & \text{if} \quad x \leq -2 \\ \frac{x - 1}{x - 2} & \text{if} \quad -2 < x < 3 \\ x & \text{if} \quad 3 < x \end{cases}$$

Problem 5._____C______ Does the function f have a vertical asymptote?

(a) No.

(b) Yes, but only at $x = 0$.

(c) Yes, but only at $x = 2$.

(d) Yes, at $x = 0$ and $x = 2$.

(e) Yes, but only at $x = -2$.

The *formula* $\frac{3}{x^2}$ has a vertical asymptote when $x = 0$; however, this input value is excluded from the set of input values where this formula is valid. The *formula*

$$\frac{x - 1}{x - 2}$$

has a vertical asymptote when $x = 2$; and this input value is included in the set of input values where this formula is valid.

Problem 6._____E_____ Does the function f have a jump discontinuity?

(a) No.

(b) Yes, but only at $x = -2$.

(c) Yes, but only at $x = 0$.

(d) Yes, at $x = -2$ and $x = 3$.

(e) Yes, but only at $x = 3$.

$$\lim_{x \to -2^-} f(x) = \frac{3}{(-2)^2} = \frac{3}{4} \quad \text{and} \quad \lim_{x \to -2^+} f(x) = \frac{-2-1}{-2-2} = \frac{3}{4}$$
$$\lim_{x \to 3^-} f(x) = \frac{3-1}{3-2} = 2 \quad \text{and} \quad \lim_{x \to 3^+} f(x) = 3$$

We must check the limit processes for the function at the endpoints of the input intervals that define the relevant domains for each formula, since these are the only input values where a jump discontinuity might occur in a piecewise defined function.

Problem 7.____A_______ Which of the following statements is true at the input value $x = 3$?

I. The function f has no output at $x = 3$.
II. We know $\lim_{x \to 3^-} f(x) = 2$. (Note that there was a typo on the Practice Exam.)
III. We know that $\lim_{x \to 3} f(x)$ does not exist.

(a) All three statements are true.

(b) Only Statement I is true.

(c) Only Statements II and III are true.

(d) Only Statement III is true.

(e) None of the statements is true.

Problem 8._____B______ What is the average rate of change for the function

$$y = f(x) = \frac{2 + 3x^{3/2}}{x}$$

on the input interval $1 \le x \le 9$?

(a) Approximately 5.00

(b) Approximately 0.528

(c) Approximately 9.22

(d) Approximately 4.22

(e) Approximately 1.153

$$\frac{f(9) - f(1)}{9 - 1} = \left(\frac{1}{8}\right)\left(\frac{2 + 3(9)^{3/2}}{9} - \frac{2 + 3(1)^{3/2}}{1}\right) = \left(\frac{1}{8}\right)\left(\frac{83}{9} - 5\right)$$

Problems 9 and 10 refer to the following scenario.

Toby is traveling toward Knoxville on Interstate 40. He used Exit 30 to get onto the highway. Let d represent the number of miles Toby has driven from Exit 30, and let t represent the number of minutes elapsed since Toby passed a rest-stop located at Mile Marker 32. Once past the rest-stop, for every increase of five miles in the value of d, the value of t increases by three minutes.

Problem 9._____E______ Which of the following statements is true, based on the definitions of the variables d and t?

I. We know $\Delta t = \frac{3}{5}\Delta d$.

II. We know that $d = f(t) = \frac{5}{3}t$.

III. We know that $d = g(t) = 2 + \frac{5}{3}t$.

(a) Only Statement I is true.

(b) Only Statement II is true.

(c) Only Statement III is true.

(d) Only Statements I and II are true.

(e) Only Statements I and III are true.

Problem 10._____C______ Eight minutes after passing the rest stop, Toby is 15.33 miles from Exit 30. What is the value of t when Toby is six miles from Exit 30?

(a) 11 minutes

(b) 5 minutes

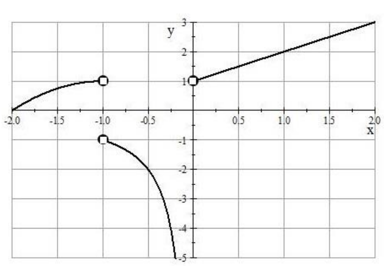
(c) Approximately 2.42 minutes

(d) Approximately 16.33 minutes

(e) Approximately -0.33 minutes

We know that $\Delta t = \frac{3}{5}\Delta d$. If we move *from* $d = 15.33$ miles *to* $d = 6$ miles, we know that $\Delta d = -9.3$ miles. Therefore, the corresponding change in time would be $\Delta t = -5.58$ minutes. Consequently, the time value that corresponds to $d = 15.33$ miles is $t = 8 - 5.58 = 2.42$ minutes.

Problems 11 – 15 refer to the graph shown below.



Problem 11. Are there any input values where the function f has a vertical asymptote? If so, what are these input values?

The function has a vertical asymptote at the input value $x = 0$.

Problem 12. Are there any input values where the function f has a jump discontinuity? If so, what are these input values?

The function has a jump discontinuity at the input value $x = -1$.

Problem 13. Are there any input values where the function f has a removable discontinuity? If so, what are these input values?

The function has no removable discontinuities.

Problem 14. Determine the numeric value of the following limiting processes, if they exist.

(a) $\lim_{x\to -1^-} f(x) = 1.0$ (b) $\lim_{x\to 0^+} f(x) = 1.0$

(c) $\lim_{x\to -1} f(x)$ Does not exist (d) $\lim_{x\to 1} f(x) = 2.0$

Problem 15. What can be said about the limiting process $\lim_{x\to 0^-} f(x)$?

This limit process does not produce a numeric value. However, there is a definite pattern to the output from the function f. In particular, we know

$$\lim_{x\to 0^-} f(x) = -\infty$$

Problem 16. Consider the function

$$y = f(x) = \frac{x^2 - 4x}{x^2 - 6x + 8}$$

Part (a). At what input values is does the function f have a discontinuity?

$$\frac{x^2 - 4x}{x^2 - 6x + 8} = \frac{x(x-4)}{(x-2)(x-4)} = \frac{x}{x-2} \quad (x \neq 4)$$

The function has discontinuities at the input values $x = 2$ and $x = 4$.

Part (b). Which of the discontinuities for the function f are removable?

The discontinuity at $x = 4$ is removable.

Part (c). If it exists, what is the numeric value of the limiting process $\lim_{x\to 3} f(x)$?

$$\lim_{x\to 3} f(x) = f(3) = 3$$

Part (d). If it exists, what is the numeric value of the limiting process $\lim_{x\to 4} f(x)$?

$$\lim_{x\to 4} f(x) = \frac{4}{4-2} = 2$$

Part (e). If it exists, what is the numeric value of the limiting process $\lim_{x\to 2} f(x)$?

This limiting process does not produce a numeric value.

Problem 17. Construct the function that gives the average rate of change for the function defined by the formula $y = f(x) = x^2 - x$ on the input interval $x = 2$ to $x = 2 + h$. Simplify your answer as much as possible.

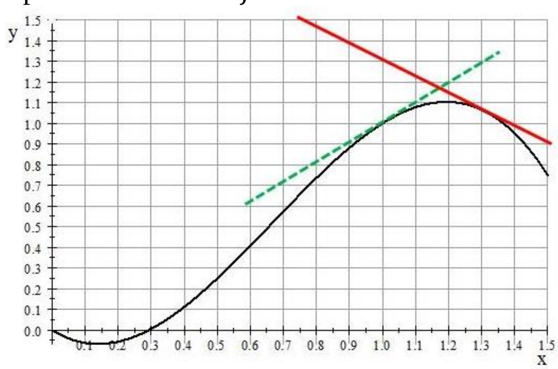
$$\begin{aligned} g_2(h) &= \frac{f(2+h) - f(2)}{h} \\ &= \frac{[(2+h)^2 - (2+h)] - [2^2 - 2]}{h} \\ &= \frac{[4 + 4h + h^2 - 2 - h] - 2}{h} \\ &= \frac{3h + h^2}{h} \\ &= \frac{\boldsymbol{h}(3+h)}{\boldsymbol{h}} \\ &= 3 + h \quad (h \neq 0) \end{aligned}$$

Problem 18. Let $r = g_2(h)$ represent the function whose formula you constructed in Problem 17. What is the numeric value of the limiting process

$$\lim_{h\to 0} g_2(h)$$

$$\lim_{h\to 0} g_2(h) = \lim_{h\to 0}(3 + h) = 3$$

Problem 19. Consider the graph of the function f shown below.



Part (a). The tangent line to the graph of the function f at the point (1.3,1.07) is shown on the diagram. Construct the formula for this tangent line.

The point (1.0,1.3) lies (approximately) on this line. Using this point along with the point of tangency, we know that the slope of the tangent line is approximately

$$m = \frac{\Delta y}{\Delta x} \approx -0.767$$

The formula for the tangent line would therefore be given by $y = T(x) \approx -0.767(x - 1.3) + 1.07$.

Part (b). Carefully draw the tangent line to the graph of f at the point $(1.0,1.0)$.

The approximate tangent line appears on the figure above.

Part (c). Use your graph to construct the formula for the tangent line you drew in Part (b).

The point (1.2,1.2) lies (approximately) on this line. Using this point along with the point of tangency, we know that the slope of the tangent line is approximately

$$m = \frac{\Delta y}{\Delta x} \approx 1.00$$

The formula for the tangent line would therefore be given by $y = T(x) \approx x$.