Problem 1._____ The function f below has a removable discontinuity at the input value $b = 0$. Which of the following functions produces the same output as the function f for all nonzero input values?

$$y = f(b) = \left(\frac{1}{b}\right)\left(\frac{1}{b^2 - 1} + \frac{1}{b^2 + 1}\right)$$

(a) $g(b) = \dfrac{1}{b-1} + \dfrac{1}{b^2+1}$

(b) $h(b) = \dfrac{1}{b-1} + \dfrac{1}{b+1}$

(c) $k(b) = \dfrac{2b}{(b^2-1)(b^2+1)}$

(d) $j(b) = \dfrac{2}{b^2-1}$

(e) $m(b) = \dfrac{1}{b-1}$

Problem 2._____ What are the input values where the function below has a removable discontinuity?

$$y = f(t) = \frac{t^2 - 1}{t^2 + 2t + 1}$$

(a) There are no removable discontinuities.

(b) Only at $t = 1$

(c) Only at $t = 1, t = 0,$ and $t = -1$

(d) Only at $t = 1$ and $t = -1$

(e) Only at $t = -1$

Problem 3._____ What are the input values where the function below has a vertical asymptote?

$$y = f(t) = \frac{t^2 - 1}{t^2 + 2t + 1}$$

(a) There are no vertical asymptotes

(b) Only at $t = 1$

(c) Only at $t = 1, t = 0,$ and $t = -1$

(d) Only at $t = 1$ and $t = -1$

(e) Only at $t = -1$

Problem 4._____ Which function below provides the average rate of change for the function $y = f(x) = 3 - 2\sqrt[3]{t}$ on the input interval from $t = 27$ to $t = 27 + h$?

(a) $g_{27}(h) = \dfrac{3+h}{h}$

(b) $g_{27}(h) = -\dfrac{6 + 2\sqrt[3]{27+h}}{h}$

(c) $g_{27}(h) = 4$

(d) $g_{27}(h) = \dfrac{6 - 2\sqrt[3]{27+h}}{h}$

(e) $g_{27}(h) = \dfrac{3 - 2\sqrt[3]{27+h}}{3 - 2\sqrt[3]{27}}$

Problems 5, 6, and 7 refer to the function whose formula is given below.

$$y = f(x) = \begin{cases} \dfrac{3}{x^2} & \text{if} \quad x \le -2 \\ \dfrac{x-1}{x-2} & \text{if} \quad -2 < x < 3 \\ x & \text{if} \quad 3 < x \end{cases}$$

Problem 5._____ Does the function f have a vertical asymptote?

(a) No.

(b) Yes, but only at $x = 0$.

(c) Yes, but only at $x = 2$.

(d) Yes, at $x = 0$ and $x = 2$.

(e) Yes, but only at $x = -2$.

Problem 6._____ Does the function f have a jump discontinuity?

(a) No.

(b) Yes, but only at $x = -2$.

(c) Yes, but only at $x = 0$.

(d) Yes, at $x = -2$ and $x = 3$.

(e) Yes, but only at $x = 3$.

Problem 7._____ Which of the following statements is true at the input value $x = 3$?

I. The function f has no output at $x = 3$.
II. We know $\lim_{x \to 3^-} f(x) = 1/2$.
III. We know that $\lim_{x \to 3} f(x)$ does not exist.

(a) All three statements are true.

(b) Only Statement I is true.

(c) Only Statements II and III are true.

(d) Only Statement III is true.

(e) None of the statements is true.

Problem 8._____ What is the average rate of change for the function

$$y = f(x) = \frac{2 + 3x^{3/2}}{x}$$

on the input interval $1 \le x \le 9$?

(a) Approximately 5.00

(b) Approximately 0.528

(c) Approximately 9.22

(d) Approximately 4.22

(e) Approximately 1.153

Problems 9 and 10 refer to the following scenario.

Toby is traveling toward Knoxville on Interstate 40. He used Exit 30 to get onto the highway. Let d represent the number of miles Toby has driven from Exit 30, and let t represent the number of minutes elapsed since Toby passed a rest-stop located at Mile Marker 32. Once past the rest-stop, for every increase of five miles in the value of d, the value of t increases by three minutes.

Problem 9._____ Which of the following statements is true, based on the definitions of the variables d and t?

I. We know $\Delta t = \frac{3}{5}\Delta d$.

II. We know that $d = f(t) = \frac{5}{3}t$.

III. We know that $d = g(t) = 2 + \frac{5}{3}t$.

(a) Only Statement I is true.

(b) Only Statement II is true.

(c) Only Statement III is true.

(d) Only Statements I and II are true.

(e) Only Statements I and III are true.

Problem 10._____ Eight minutes after passing the rest stop, Toby is 15.33 miles from Exit 30. What is the value of t when Toby is six miles from Exit 30?

(a) 11 minutes

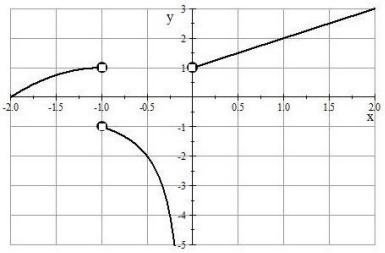
(b) 5 minutes

(c) Approximately 2.42 minutes

(d) Approximately 16.33 minutes

(e) Approximately -0.33 minutes

Problems 11 – 15 refer to the graph shown below.



Problem 11. Are there any input values where the function f has a vertical asymptote? If so, what are these input values?

Problem 12. Are there any input values where the function f has a jump discontinuity? If so, what are these input values?

Problem 13. Are there any input values where the function f has a removable discontinuity? If so, what are these input values?

Problem 14. Determine the numeric value of the following limiting processes, if they exist.

(a) $\lim_{x\to -1^-} f(x)$ (b) $\lim_{x\to 0^+} f(x)$ (c) $\lim_{x\to -1} f(x)$ (d) $\lim_{x\to 1} f(x)$

Problem 15. What can be said about the limiting process $\lim_{x\to 0^-} f(x)$?

Problem 16. Consider the function

$$y = f(x) = \frac{x^2 - 4x}{x^2 - 6x + 8}$$

Part (a). At what input values is does the function f have a discontinuity?

Part (b). Which of the discontinuities for the function f are removable?

Part (c). If it exists, what is the numeric value of the limiting process $\lim_{x\to 3} f(x)$?

Part (d). If it exists, what is the numeric value of the limiting process $\lim_{x\to 4} f(x)$?

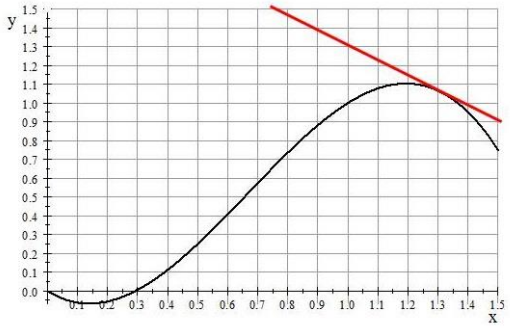
Part (e). If it exists, what is the numeric value of the limiting process $\lim_{x\to 2} f(x)$?

Problem 17. Construct the function that gives the average rate of change for the function defined by the formula $y = f(x) = x^2 - x$ on the input interval $x = 2$ to $x = 2 + h$. Simplify your answer as much as possible.

Problem 18. Let $r = g_2(h)$ represent the function whose formula you constructed in Problem 17. What is the numeric value of the limiting process

$$\lim_{h\to 0} g_2(h)$$

Problem 19. Consider the graph of the function f shown below.



Part (a). The tangent line to the graph of the function f at the point $(1.3,1.07)$ is shown on the diagram. Construct the formula for this tangent line.

Part (b). Carefully draw the tangent line to the graph of f at the point $(1.0,1.0)$.

Part (c). Use your graph to construct the formula for the tangent line you drew in Part (b).