



**Problem 4.** Suppose that the function  $y = f(x)$  satisfies the conditions of the Mean Value Theorem on the input interval  $-2 \leq x \leq 3$ , and suppose that  $f(-2) = 4$  and  $f(3) = 8$ . Which of the following statements must be true for at least one input value  $x = a$  in this input interval?

(a)  $f'(a) = 4$

(b)  $f'(a) = 5$

(c)  $f'(a) = \frac{4}{5}$

(d)  $f'(a) = -2$

(e)  $f'(a) = \frac{8}{3}$

Let  $y = f(x)$  be a function whose derivative function is given by the formula below. Problems 5 and 6 refer to this function.

$$y = f'(x) = \begin{cases} \frac{3}{x^2} & \text{if } x < -2 \\ x & \text{if } -2 < x < 3 \\ -x & \text{if } 3 < x \end{cases}$$

**Problem 5.** What is the formula for the second derivative function  $a = f''(x)$  on the input interval  $x < -2$ ?

(a) The second derivative cannot be computed.

(b)  $f''(x) = 1$

(c)  $f''(x) = -\frac{2}{(x-2)^2}$

(d)  $f''(x) = -\frac{6}{x^3}$

(e)  $f''(x) = \frac{3}{2x}$

**Problem 6.** Which of the following statements is true?

- I. The function  $f$  does not have any local extrema.
- II. The function  $f$  has a local maximum output at the input value  $x = 3$ .
- III. The function  $f$  has a local minimum output at the input value  $x = 2$ .

(a) Only Statement I is true.

(b) Only Statement II is true.

(c) Only Statement III is true.

(d) Statements II and III are both true.

(e) None of these statements is true.

**Problem 7.** \_\_\_\_\_ If  $y = f(x) = \sqrt{x}$ , then what is the formula for  $a = f''(x)$ ?

(a)  $f''(x) = -\frac{1}{4}x^{-3/2}$

(b)  $f''(x) = \frac{3}{x\sqrt{x}}$

(c)  $f''(x) = \frac{1}{2\sqrt{x}}$

(d)  $f''(x) = 3x$

(e)  $f''(x) = 2x^{3/2}$

**Problem 8.** \_\_\_\_\_ Suppose we know that the point  $(2, -4)$  lies on the graph of a function  $y = f(x)$ , and suppose we also know that  $f'(2) = 5$ . What is the formula for the tangent line to the graph of  $f$  at the point  $(2, -4)$ ?

(a)  $y - 5 = -4(x - 2)$

(b)  $y = 2(x - 5) - 4$

(c)  $y = 5(x - 2) - 4$

(d)  $y - 2 = 5(x + 4)$

(e)  $y = 4(x + 2) - 5$

**Problem 9.** Consider the function  $y = f(x) = 3x^{-1}$ .

**Part (a).** Construct the formula for the function  $r = g_4(h)$  that gives the average rate of change for the function  $f$  on the input interval from  $x = 4$  to  $x = 4 + h$ .

**Part (b).** Use the formula for the function  $g_4$  and the limit definition of the derivative to compute the value of  $f'(4)$ .

**Problem 10.** Let  $y = f(x) = 4x^3 - x^{1/3}$ . Use the Sum Rule and the Constant Multiple Rule along with the specific derivative formulas we have developed to construct the formula for  $r = f'(x)$ .

**Problem 11.** Let  $y = f(x) = x^2 \cdot 3^x$ . Use the Product Rule along with the specific derivative formulas we have developed to construct the formula for  $r = f'(x)$ .

**Problem 12.** Consider the function

$$y = f(x) = \frac{x^2 - 3}{4 - x^2}$$

**Part (a).** Use the Quotient Rule and the specific derivative formulas we have developed to construct the formula for  $r = f'(x)$ .

**Part (b).** Construct the formula for the tangent line to the graph of the function  $f$  at the point  $(-1, f(-1))$ .

**Problem 13.** Consider the function  $y = f(x) = 2x^3 + 3x^2 - 12x + 5$ .

**Part (a).** Use the Sum and Constant Multiple Rules, along with the special derivative formulas we have developed to construct the formula for  $r = f'(x)$ .

**Part (b).** At what input values will the tangent line to the graph of  $f$  be horizontal?

**Part (c).** At what input values will the tangent line to the graph of  $f$  have slope  $m = -12$ ?

**Part (d).** What is the formula for the tangent line to the graph of  $f$  at the point  $(0, f(0))$ ?

**Problem 14.** Consider the function  $f(x) = 4x^{-1}e^x$ . What is the formula for the function  $a = f''(x)$ ?

**Problem 15.** The graph of a function  $y = f(x)$  is shown below. On the grid provided, sketch the graph of the function  $a = f''(x)$ .

