ALGEBRA UNIT 1 — Getting to Know the Terms

Before we get started, we should make a few quick defnitions. In these notes, we will be using the word "variable", "unknown", "constant", "expression", and "equation" quite a bit; so it would be a good idea to make sure everyone understands what these words mean.

- A *variable* is a symbol representing numeric values of some quantity in a particular problem that might be able to change. Variables are usually denoted by Roman or Greek letters.
- A *constant* is a number that does not change in a specific problem. Constants can also be denoted by letters.
- An *expression* is a collection of numbers and variables tied together by operations like addition, multiplication, etc.
- An *equation* is a pair of expressions connected by an equal sign.[1]
- An *inequality* is a pair of expressions linked by one of the symbols $\leq$, $<$, $\geq$, or $>$. The symbol always points toward the smaller of the two expressions.
- A variable in an equation or inequality becomes an *unknown* when we are trying to identify specific values for it that make the equation or inequality true. Unknowns can appear only in equations or inequalities.

Expressions are themselves built from other expressions; consequently, we have a lot of terms used to refer to parts of an expression. These terms are not always used consistently and sometimes can be confusing; unfortunately, there is no way to avoid this. In these notes, however, we will be as consistent as possible. Here are some words and phrases commonly used to refer to parts of expressions.

- TERM

Individual expressions added together to create a larger expression are often called *terms* of the bigger expression. For example, the expression

$$3x - 2t + 5$$

contains three terms, namely $3x$, $-2t$, and 5.

- FACTOR

Individual expressions multiplied together to create a larger expression are often called *factors* of the bigger expression. There are often many ways to break an expression into factors. For example, the expression

$$3xt^2$$

has 3, x, and t^2 as factors. However, it also has $3xt$ and t as factors, along with many others. How an expression is broken into factors is a matter of convenience.

- COEFFICIENT

When we are focusing on a particular factor in a product of expressions, all of the other factors in that expression together form the *coefficient* of the particular factor we are looking at. For example, the coefficient of t^2 in the expression

$$3xt^2$$

is the factor $3x$. The coefficient of x in this expression would be $3t^2$.

[1] This understanding of the word "equation" is inadequate in more advanced mathematics but will suffice for now.

- LIKE TERMS

When two or more terms in a sum contain the same factor, we refer to them as *like terms* with respect to this particular factor. The particular factor we are focusing on is often called a *common factor* of the like terms. For example,

$$3x + \sqrt{2}t^2 + 4xy + 5t^2$$

contains two pairs of like terms, namely $3x$ and $4xy$ (like terms with respect to x), and $\sqrt{2}t^2$ and $5t^2$ (like terms with respect to t^2). We would say that $3x$ and $4xy$ have x as a common factor; likewise, $\sqrt{2}t^2$ and $5t^2$ have t^2 as a common factor. It is customary to rewrite like terms as a single term by addng together their coefficients. For example,

$$3x + \sqrt{2}t^2 + 4xy + 5t^2 = (3 + 4y)\,x + \left(\sqrt{2} + 5\right)t^2$$

This process is called *combining like terms*. It is worth noting that $3x$ and $4xy$ are *not* like terms with respect to y. Constant terms (terms that do not contain a variable) are always considered to be like terms. We will have more to say about this in Unit 3.

Example 1 *Combine any like terms in the expression* $\dfrac{3y^3 - 7y^3}{5xp + p^2}$.

Solution. The numerator of this expression contains two like terms with respect to y^3. Thus, we know that

$$\frac{3y^3 - 7y^3}{5xp + p^2} = \frac{(3-7)\,y^3}{5xp + p^2} = \frac{-4y^3}{5xp + p^2}$$

The denominator of this expression does not contain any like terms with respect to x or with respect to p^2. However, it *does* contain two like terms with respect to p. Observe that

$$5xp + p^2 = [5x]\,p + [p]\,p = (5x + p)\,p$$

Consequently, we can also write

$$\frac{3y^3 - 7y^3}{5xp + p^2} = \frac{-4y^3}{(5x + p)\,p}$$

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Example 2 *Rewrite the following expression, first by combining like terms with respect to t, then by combining like terms with respect to $\sqrt{a}$.*

$$2\sqrt{a}t + 7xt - 4\sqrt{a} + 5yt$$

Solution. There are three like terms with respect to t. Adding the coefficients of these terms, we see that

$$2\sqrt{a}t + 7xt - 4\sqrt{a} + 5yt = \left(2\sqrt{a} + 7x + 5y\right)t - 4\sqrt{a}$$

There are two like terms with respect to $\sqrt{a}$. Adding the coefficients of these terms, we see that

$$2\sqrt{a}t + 7xt - 4\sqrt{a} + 5yt = (2t - 4)\sqrt{a} + +7xt + 5yt$$

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In the previous example, one form of the expression is really not superior to the others. How like terms are combined often depends on how the expression will be used.

Example 3 *Are there any variables in the expression* $\dfrac{3x - 5t^3}{y}$*?*

Solution. This is actually a tricky question. There are three *letters* in this expression, but are they *variables*? In other words, do any of the letters in this expression represent possible values for changing quantities? Without some additional information, we cannot be sure. This expression is *decontextualized* — it does not come with any information. In a decontextualized expression, we assume every letter represents a variable. Thus, we would *assume* this expression contans three variables.

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Example 4 *Are there any unknowns in the expression* $\dfrac{3x - 5t^3}{y}$*?*

Solution. No. Unknowns can only appear in equations or inqualities. This expression contains three *variables* but cannot contain unknowns.

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Example 5 *Are there any unknowns in the equation* $A = xy$*?*

Solution. No. We have a decontextualized equation, and there are three variables in this equation. However, we are not asked to identify specific values for these variables, so none of them serve as unknowns.

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Example 6 *If a circle has radius* R*, then the area* A *of this circle is given by the equation* $A = \pi R^2$*, where* $\pi \approx 3.14159$*. How many variables are in this equation? Are there any unknowns in this equation?*

Solution. This is an example of a *contextualized* equation — we are provided information to go along with it. In particular, we are told that both A and R represent values of quantities that can change for circles, namely the radius and the area. Consequently, we know that A and R serve as variables in this equation. We are also told that the letter π has a specific value (approximately 3.14159). Consequently, the letter π serves as a *constant* in this equation and not a variable. Thus, this contextualized equation has two variables. Since we are not asked to identify specific values for these variables, neither of them serve as unknowns.

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Note: In the sciences, certain constants appear so often that they are assigned special letter names. Whenever these special letters appear in an expression, we usually assume the letter represents that constant, even if the expression is decontextualized. The ratio between the radius and circumference of a circle is one of these constants. This ratio is assigned the special letter π which is the first letter in the Greek work *perimetros*. As mentioned in the last example, $\pi \approx 3.14159$. *Euler's constant* (approximately 2.71828) is another example. This constant is denoted by e and appears frequently in business and physics applications. With the exception of π and e, most of these special constants do not appear in elementary mathematics.

Example 7 *If a circle has radius R, then the area A of this circle is given by the equation $A = \pi R^2$. Are there any unknowns in this equation if we want to find the area of a circle when its radius is three inches?*

Solution. Yes. We want to find the value of the variable A when the radius is three inches. Consequently, A serves as an unknown in this equation. Both π and R serve as constants since they are assigned specific values.

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Example 8 *Consider the inequality $3x + 2y > 0$. In which of the following situations does y serve as an unknown?*

1. *Solve the inequality $3x + 2y > 0$ for y.*
2. *What values of y make the inequality $3x + 2y > 0$ true when $x = 1$?*

Solution. A variable serves as an unknown in an equation or inequality only when we are asked to find specific values for it that make the equation or inequality true. With this in mind, y serves as an unknown only in the second situtation.

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Exercises for Unit 1

(1) In the expression $4\sqrt{7}xyt$, determine the coefficient of x, the coefficient of y, and then the coefficient of t.

Determing the coefficient of b in the following expressions. You will need to combine like terms.

(2) $7xtb - 2yb$ (3) $\frac{2}{3}b - \frac{4}{5}b$

Each of the following expressions contains two or more like terms with respect to y^2. Rewrite each expression by combining these like terms.

(4) $6ty^2 - 2ty^2$ (5) $\sqrt{3}y^2 + 5\pi y^2 - 4x$ (6) $7xy^3 + 4py^2 - 9xy$ (7) $\dfrac{5\sqrt{2}y^2 + 3y^2 - x}{xy^2 + t + 2xy^2}$

Rewrite the following expressions, first by combining like terms with respect to x, then by combining like terms with respect to p^3.

(8) $2xy + 7xp^3 - 3p^3$ (9) $xp^3 - 2\pi x^2 p^3 + 8$ (10) $7xp^4 + 4px^2 - 9xp^3$ (11) $\dfrac{xp^3 - 2xp^4 + 5x}{p^3 + xp^3 - x}$

(12) The volume V of a cylinder is given by the equation $V = \pi R^2 H$, where R is the radius of the base of the cylinder and H is its vertical height. How many variables are in this equation? If we know the volume of a cylinder is 30 cubic feet and its height is 4 feet, what is the unknown in this equation? (You are not being asked to solve for the value of the unknown here, only to identify it.)

(13) The magnitude E of the electric field (measured in Newtons per coulomb) generated by a particle having charge Q coulombs at a distance R meters from the particle is given by the equation

$$E = \frac{1}{4\pi\epsilon_0}\left(\frac{|Q|}{R^2}\right)$$

where $\epsilon_0 \approx 8.8542 \times 10^{-12}$ Farads per meter is the *vacuum permittivity*. What are the constants in this equation? If we know the electric field has magnitude .03 Newtons per coulomb at a distance of .01 meters from the particle, what is the unknown in this equation?