PATHWAYS THROUGH CALCULUS

An Inquiry-Oriented Calculus I Initiative at Middle Tennessee State University

In 2014, MTSU began using *Pathways to Calculus: A Problem Solving Approach* (Carlson, Oehrtman, and Moore) as its precalculus text.

Two distinguishing philosophical features of this text:

1. The book is designed to support a *sense-making* pedagogy described in the classroom Rules of Engagement:
   - **Speak with meaning**
     --- What you say should carry meaning to others
     --- Use quantities, not pronouns
     --- Explain and justify your approach
   - **Exhibit intellectual integrity** --- Ask questions; don't pretend to understand something you don't
   - **Strive to make sense** --- Persist in working to understand your peers' thinking
   - **Respect the learning process of others**
     --- Allow others time to think, reflect, and construct meaning
     --- Pose questions to help others construct meaning

2. The text develops precalculus content understanding while focusing on

- **Careful definition of, and reasoning with, quantities**
- **The concept of function as a dynamic process with careful attention to notation and terminology**
- **Development of *covariational reasoning* skills** --- holding in mind a sustained image of the way the measures of two quantities vary simultaneously and coordinating how change in the measure of one quantity affects change in the measure of other

After meeting with M. Carlson in Spring 2016 to discussing strategy, I set out to create a Calculus I course that focuses on these philosophical features.

To date, twenty-six investigations have been developed and class-tested over four semesters (Spring 2017 – Fall 2018).

Other projects have also utilized this philosophy, notably

- Thompson, et al. --- Project DIRACC (Newton Meets Technology)
- M. Oehrtman, et. al. --- Project CLEAR Calculus

Very different approaches to content development

Potential drawback for these projects --- Heavy reliance on technology

The design of the investigations is informed by Carlson's Covariation Framework:

The following mental actions are associated with the development of covariational reasoning skills:

- (MA1) Students have an image of two variables changing simultaneously.

- (MA2) Students have a loosely coordinated image of how the variables are changing with respect to each other.

- (MA3) Students have an image of the amount of change in one variable while considering changes in discrete amounts of the other variable.

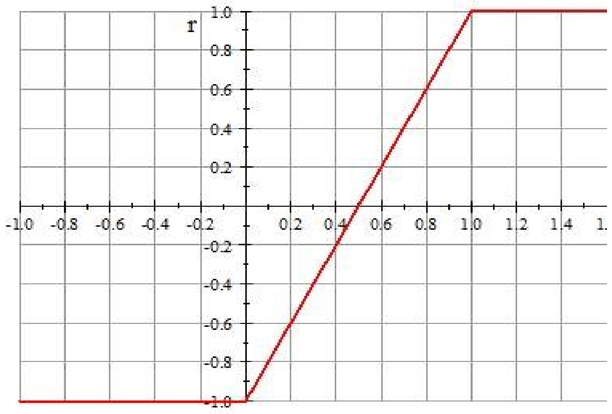
- (MA4) Students have an image of average rate-of-change / slope for contiguous input intervals of the function's domain.

- (MA5) Students have an image of continuously changing rate-of-change for a function over its domain

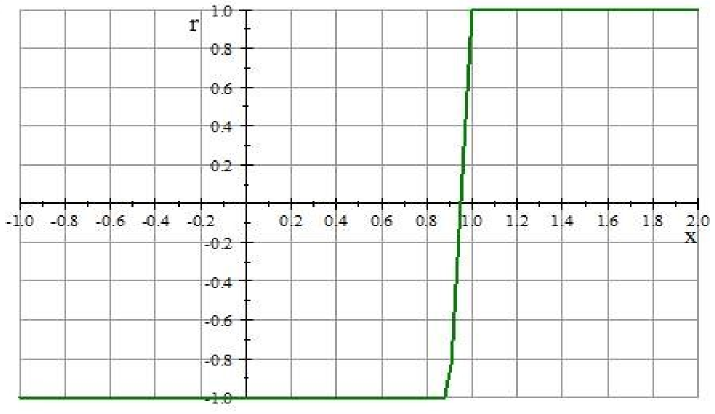
Excerpt from Investigation 8

Theme of Investigation 8: Students reason from average rate of change functions to develop intuition for when a function fails to be differentiable at an input value.

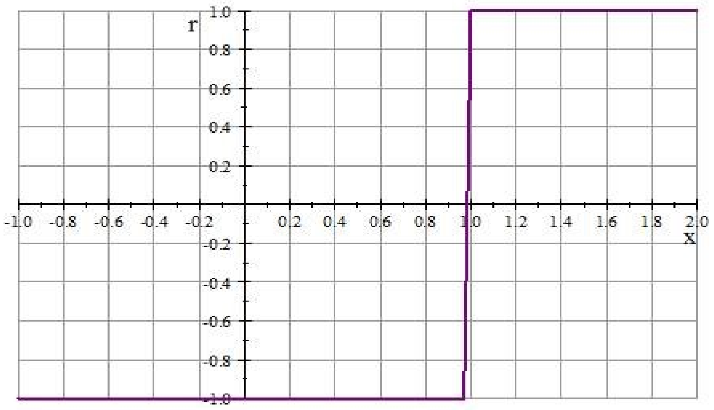
The diagram below shows the graphs of three average rate of change functions for the function $y = f(x) = 1 + |x - 1|$.



Graph of ARC_1



Graph of $\mathrm{ARC}_{0.1}$



Graph of $\mathrm{ARC}_{0.01}$

- Based on these graphs, what do you think the value of $f'(1.6)$ should be? Explain your thinking.
- Based on these graphs, what do you think the value of $f'(0.8)$ should be? Explain your thinking.
- Based on these graphs, what do you think the value of $f'(1)$ should be? Explain your thinking.
- Consider the function $y = f(x) = 1 + |x - 1|$

Part (a). Use algebra to show that

$$\mathrm{ARC}_h(1) = \frac{|h|}{h} = \begin{cases} 1 & \text{if} \quad h > 0 \\ -1 & \text{if} \quad h < 0 \end{cases}$$

Part (b). Think about the formula in Part (a). If we consider only *positive* values of h, what can we say about the value of $\mathrm{ARC}_h(1)$ as the values of h get closer and closer to 0? What would you predict $f'(1)$ should be?

Excerpt from Investigation 15

First Corollary to the Mean Value Theorem

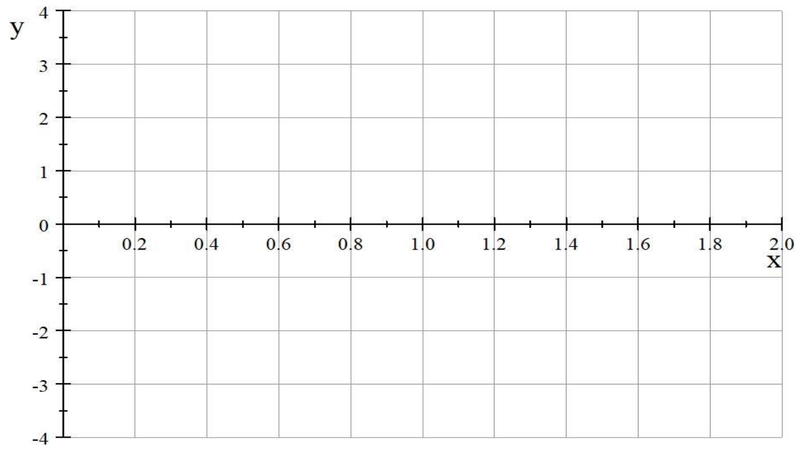
Let $y = f(x)$ be a differentiable function.

(1) If $f'(x) > 0$ for all input values x in some open interval $a < x < b$, then the output of the function f is increasing on this interval.

(2) If $f'(x) < 0$ for all input values x in some open interval $a < x < b$, then the output of the function f is decreasing on this interval.

- Assume that $y = f(x)$ is any function that is differentiable on the input interval $0 \leq x \leq 2$, and suppose $r = f'(x)$ has positive output on this interval.

- Choose any two input values $x = u$ and $x = v$ you like on the input axis below --- only make sure $u < v$.



- We have assumed that f is differentiable on the input interval $0 \leq x \leq 2$. Explain why this guarantees that $f(u)$ and $f(v)$ exist.

- Choose any value for $f(u)$ and $f(v)$ you like --- so long as both choices are between -4 and 4. Draw the points $(u, f(u))$ and $(v, f(v))$ on the grid.

- Explain why we know the function f *must* be continuous in the interval $[u, v]$.

- Draw the straight line segment that passes through the two points $(u, f(u))$ and $(v, f(v))$. What does the slope of this line segment represent in the context of this problem?

- Explain why we know there is some input value $x = a$ in your interval such that $f'(a)$ equals the slope of your line segment.

- We are *assuming* that $f'(a) > 0$. Is it possible to have $f'(a) > 0$ *and* be equal to the slope of your line segment? If not, what would you need to change about your choice of $f(v)$ in order to make it possible?

Data has been collected comparing the inquiry-oriented sections of Calculus I to the other lecture-based sections.

DATA GATHERING ISSUES

- No common final exam in Calculus I; therefore, no easy way to compare content knowledge
- DFW-rates vary greatly across Calculus I sections; therefore no easy way to compare student success
- I am only instructor using the Pathways through Calculus materials

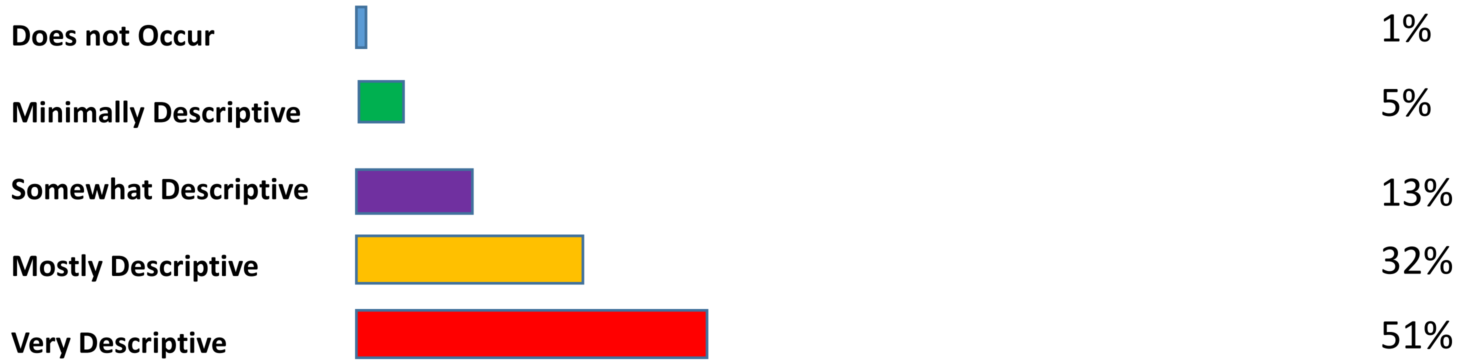
DATA GATHERING STRATEGY

- Use PIPS (Post-Instruction Practices Survey) to compare student perceptions
- Compare persistence and student success in Calculus II

Selected PIPS Survey Results for Fall 2018

Lecture-Based Classes

My instructor guides me through major topics as I listen and take notes.



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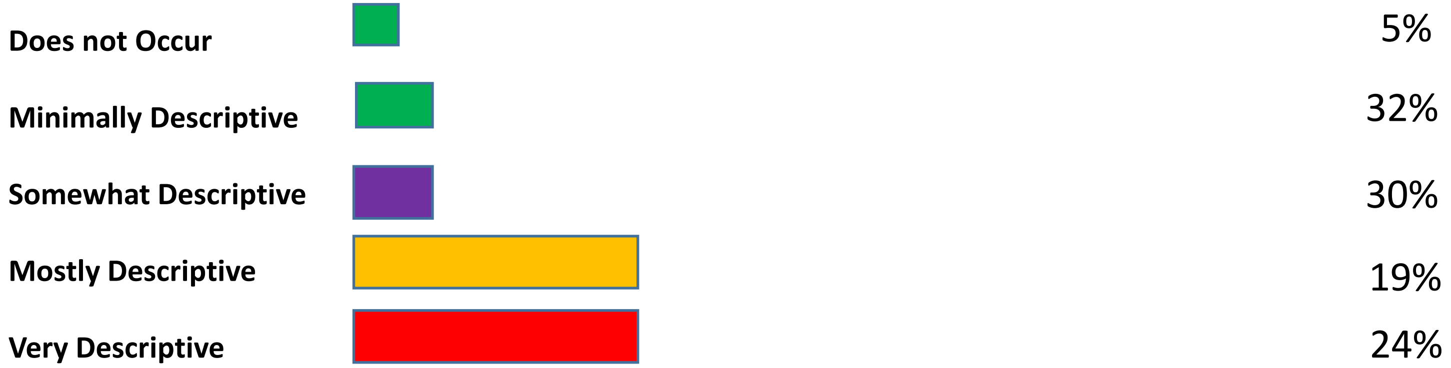

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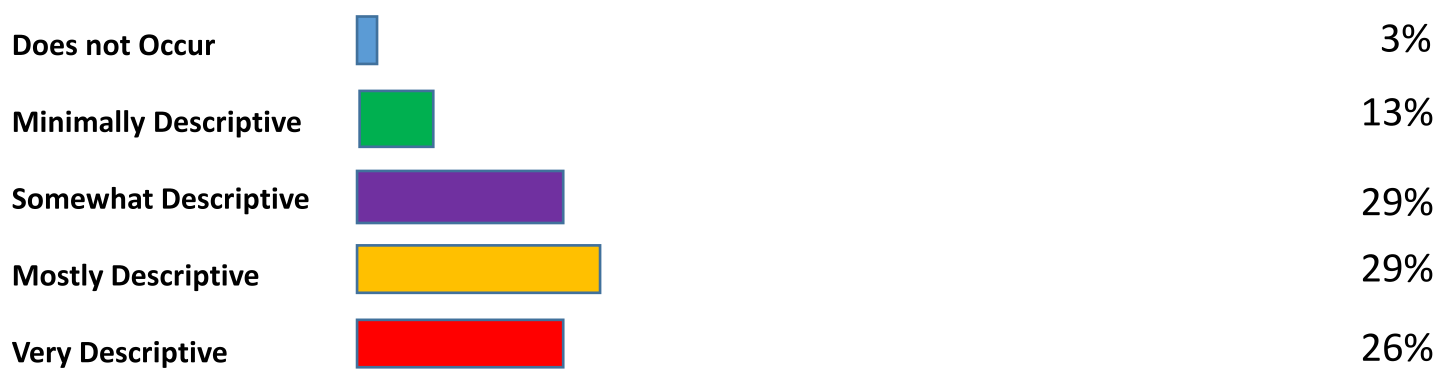

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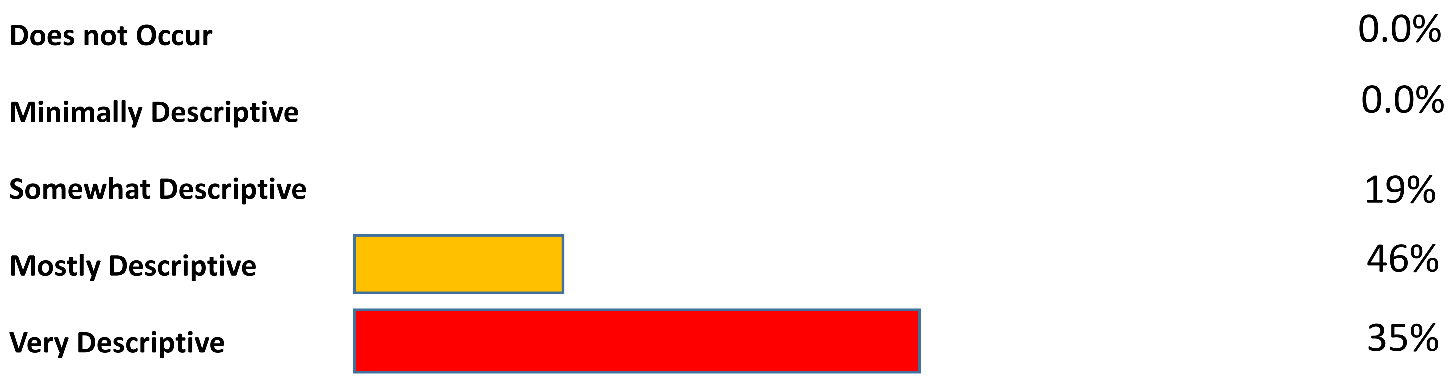

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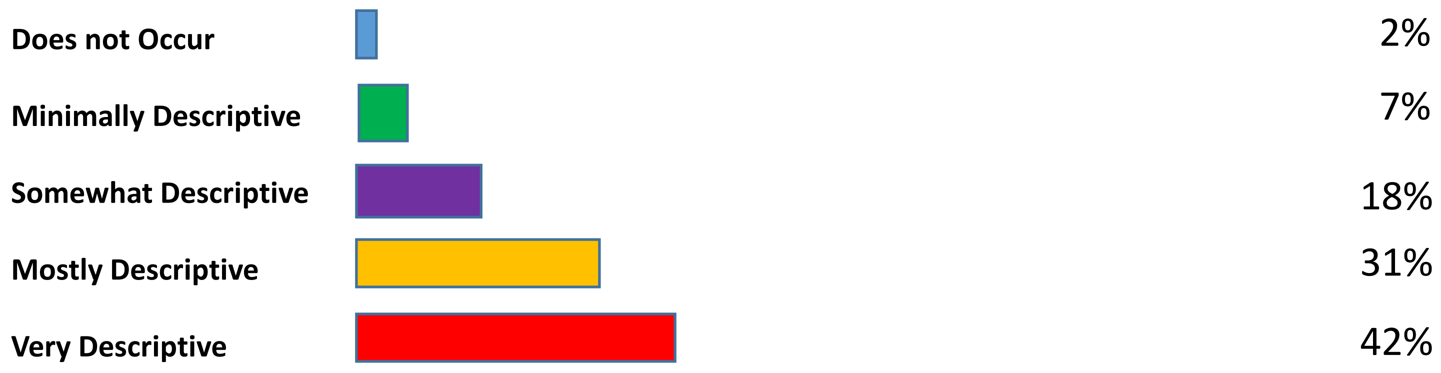

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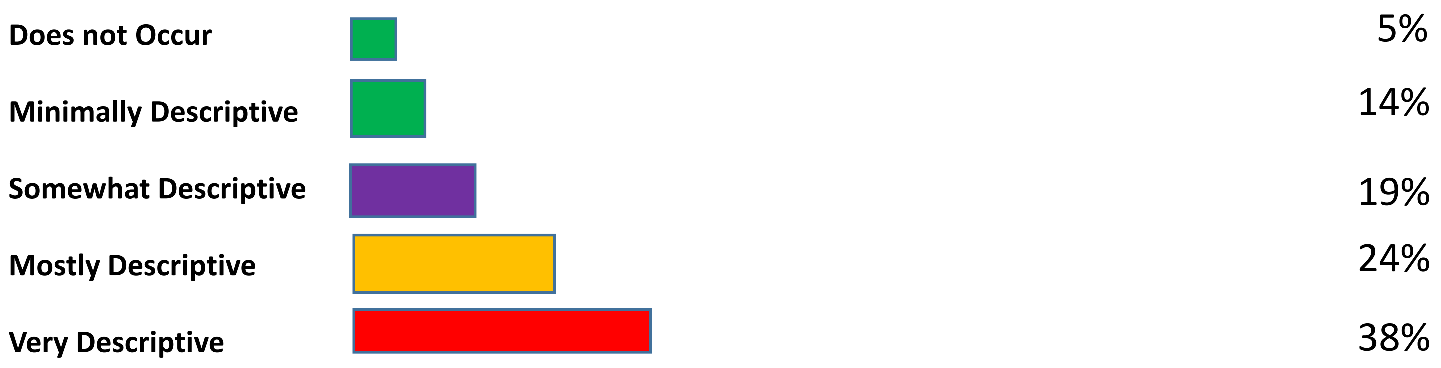

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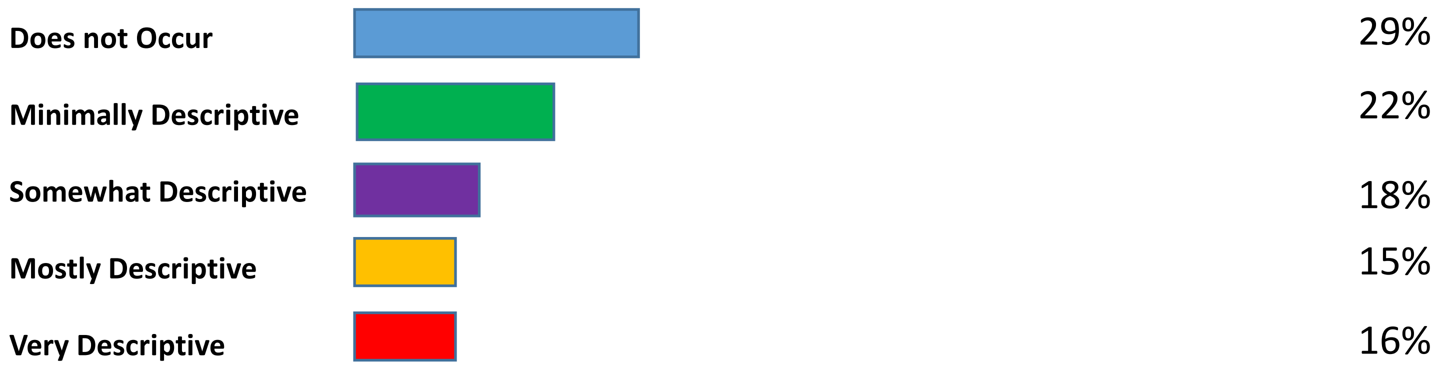

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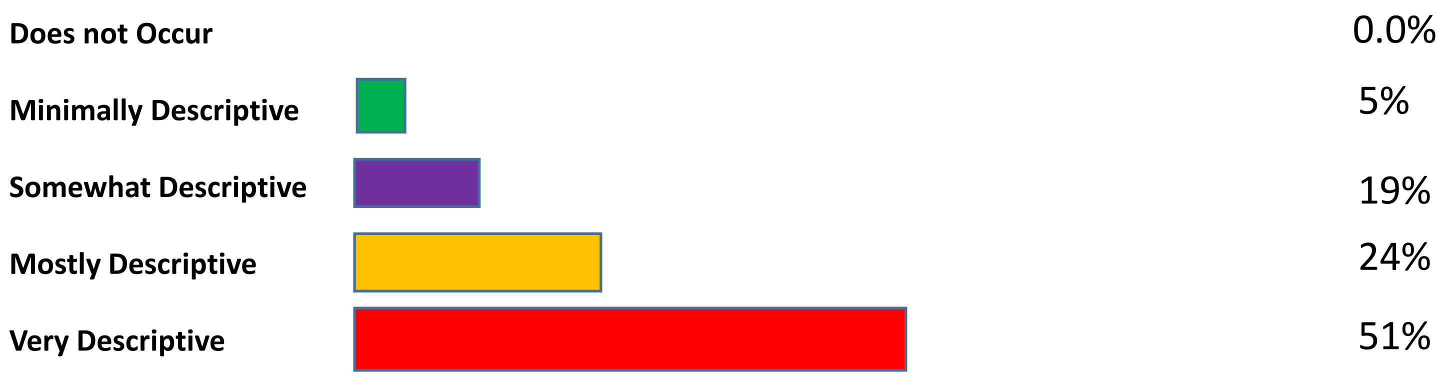

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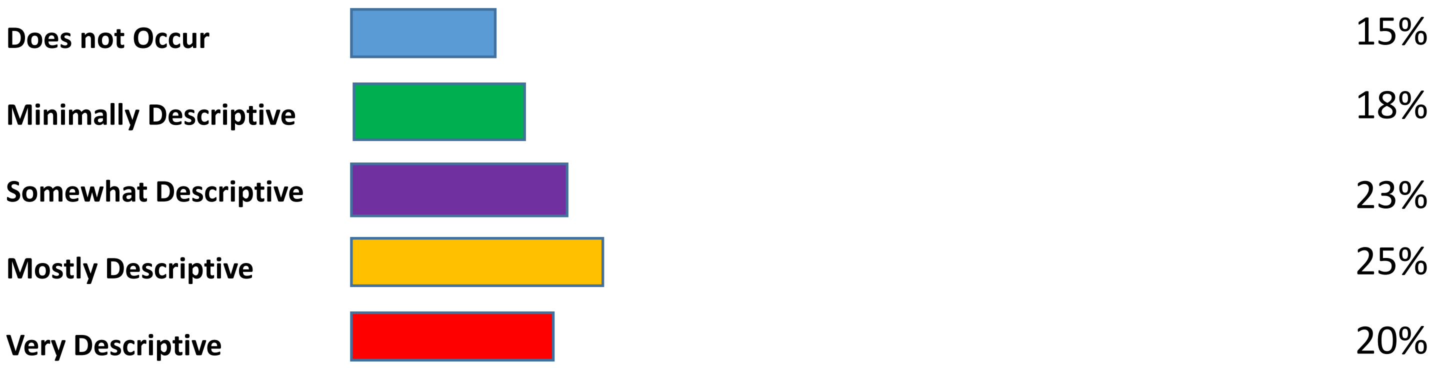

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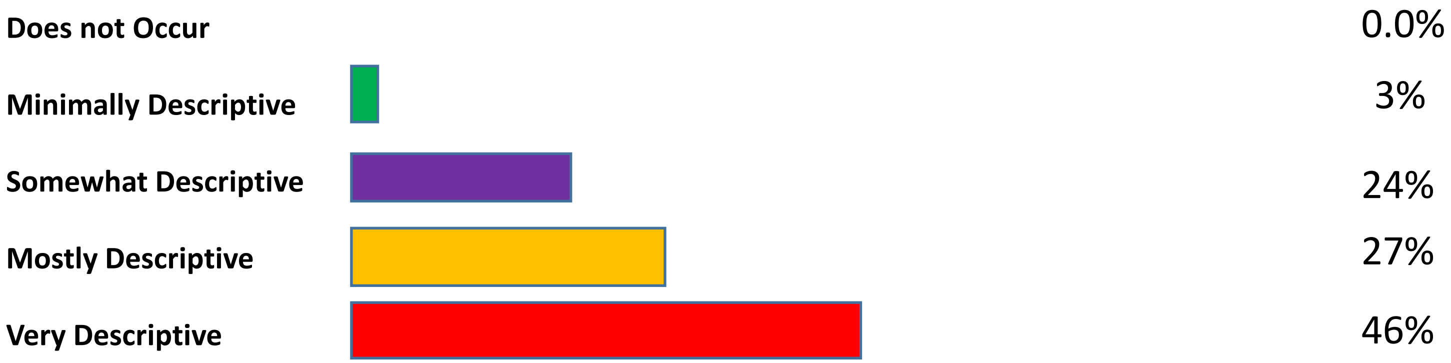

Lecture-Based Classes

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Pathways Through Calculus Classes

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