Problem 1. A car is driving away from a crosswalk. Let d represent the measures of the quantity "the distance between the car and the crosswalk", measured in feet from the crosswalk. Let t represent the measures of the quantity "the time passed since the car started moving", measured in seconds. Suppose we know that the values of d and t are related by the function

$$d = f(t) = 3.5 + t^2$$

Part (a). Fill in the table below.

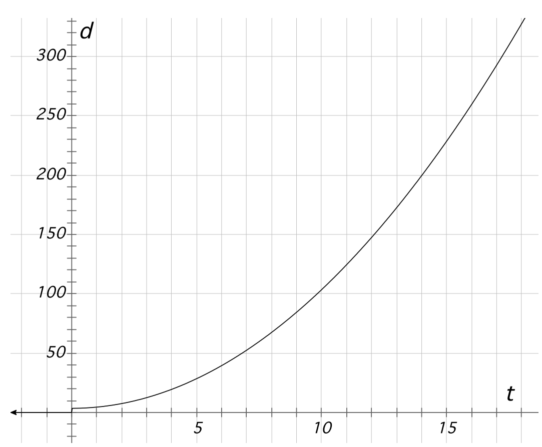
Δt	Value of t	Value of $f(t)$	$\Delta f(t)$
	0		
	0.75		
	2.25		
	3.00		
	4.50		
	5.25		

Does the car's distance from the crosswalk vary at a constant rate with respect to the number of seconds elapsed since the car started moving? Justify your response using the meaning of constant rate of change.

Part (b). A second car traveling at a constant rate of speed passed the first car the moment it started moving (at $t = 0$ seconds). The first car passed the second car 17 seconds later.

i. At what constant speed was the second car traveling?

ii. Below is a graph of the relationship between the first car's distance d (in feet) from the crosswalk and the number of seconds t elapsed since the first car started moving. Illustrate on this graph the constant speed of the second car computed in Part (i) from $t = 0$ to $t = 17$ seconds. Explain how what you drew illustrates the second car's constant rate of change.

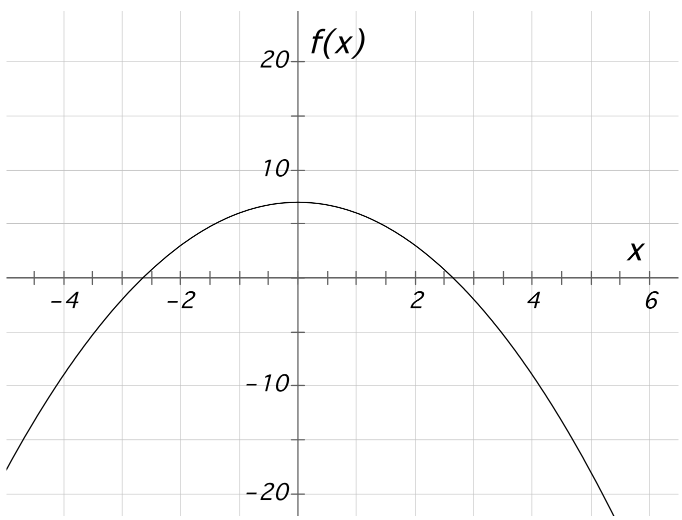


Problem 2. Suppose the values of x and y are related by the function $y = f(x) = -x^2 + 7$.

Part (a). Do the values of $f(x)$ vary at a constant rate with respect to x? Justify your response using the meaning of constant rate of change.

Part (b). Now, suppose that $y = S(x)$ is a *linear* function defined on the input interval $-3 \leq x \leq 5$. If $\Delta S(x) = \Delta f(x)$ as the values of x change from $x = -3$ to $x = 5$, then what is the constant rate of change of the linear function S?

Part (c). Below is a graph of the function f. Draw the graph of the linear function S on this same grid. Explain how what you drew illustrates a constant rate of change.



Average Rate of Change for a Function on an Input Interval

Suppose that $y = f(x)$ is a function that is defined on an input interval $a \leq x \leq b$. The *average rate of change* for the function f with respect to x on the input interval $a \leq x \leq b$ is defined to be the constant rate of change for the linear function $y = S(x)$ with the property that $S(a) = f(a)$ and $S(b) = f(b)$.

The constant rate of change for the linear function S is given by the formula

$$m = \frac{f(b) - f(a)}{b - a}$$

The graph of the linear function S is called the *secant line* for the graph of the function f on the input interval $a \leq x \leq b$.

Problem 3. Suppose that the values of x and y are related by the function $y = f(x) = -x^2 + 7$. Write an expression that represents the average rate of change of the function f on the input interval $a \le x \le b$.

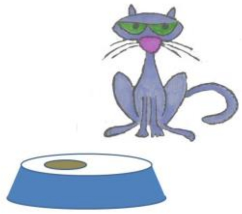
Problem 4. Justo is trying to give Mr. Hipsickles a fish oil supplement, and a drop of the fish oil lands in his water bowl. The fish oil spreads across the surface of the water as a circular disk. Let A represent the possible measures of the area (measured in square millimeters) for this disc. Let t represent the measures of the time passed since the disk began to spread, measured in seconds; and suppose we know

$$A = f(t) = \pi(7.84t)^2$$

Part (a). Describe the meaning of each of the following expressions in the context of the situation.

$$f(t+3) \qquad\qquad f(t+3) - f(t) \qquad\qquad \frac{f(t+3) - f(t)}{(t+3) - t}$$

Part (b). Evaluate $\frac{f(t+3)-f(t)}{(t+3)-t}$ for $t = 0.5$ seconds. Describe the meaning of this value in the context of this problem.



Difference Quotient

Let $y = f(x)$ be a function and let $x = a$ be an input value for the function f. The *difference quotient* function

$$r = g_a(h) = \frac{f(a+h) - f(a)}{(a+h) - a} = \frac{f(a+h) - f(a)}{h}$$

gives the average rate of change for the function f with respect to x on the input interval from $x = a$ to $x = a + h$.

Problem 5. Let P represent the measures of the quantity “the population of Telluride Colorado”, measured in thousands; and let t represent the measures of the quantity “the time passed since January 1, 1990”, measured in years. Suppose we know that the values of P are related to the values of t according to the function

$$P = f(t) = 1.645 \cdot (1.06)^t$$

Part (a). Construct the formula for the function $r = g_0(h)$ that determines the average rate of change of Telluride’s population over time intervals that begin or end at $t = 0$ years (that is, January 1, 1990).

Part (b). Construct the formula for the function $r = g_{0.203}(h)$ that determines the average rate of change of Telluride’s population over time intervals that begin or end at $t = 0.203$ years (that is, March 15, 1990).

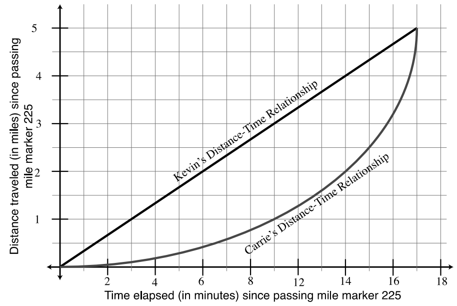
Part (c). In the context of this problem, explain the meaning of output from the function

$$r = j(t) = \frac{f(t + 0.203) - f(t)}{0.203}$$

Part (d). How is the meaning of the output from the function j different from the meaning of the output from the function $g_{0.203}$?

Homework.

Problem 1. The following graph represents the distance-time relationship for Kevin and Carrie as they cycled on a road from mile marker 225 to mile marker 230.



a. How does the distance traveled and time elapsed compare for Carrie and Kevin as they traveled from mile marker 225 to mile marker 230?

b. How do Carrie and Kevin's speeds compare as they travel from mile marker 225 to mile marker 230?

c. How do Carrie and Kevin's average speeds compare over the time interval as they traveled from mile marker 225 to mile marker 230?

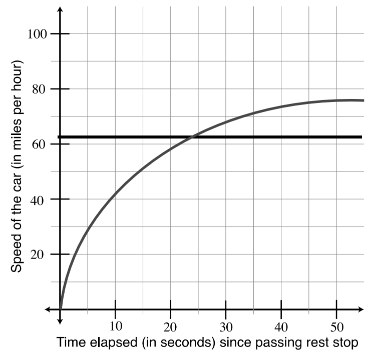
Problem 2. On a trip from Tucson to Phoenix via Interstate 10, you used your cruise control to travel at a constant speed for the entire trip. Since your speedometer was broken, you decided to use your watch and the mile markers to determine your speed. At mile marker 219 you noticed that the time on your digital watch just advanced to 9:22 AM. At mile marker 197 your digital watch advanced to 9:46 AM.

a. Compute the constant speed at which you traveled over the time period from 9:22 AM to 9:46 AM.

b. As you were passing mile marker 219 you also passed a truck. The same truck sped by you at exactly mile marker 197. Construct a distance-time graph of your car. On the same graph, construct one possible distance-time graph for the truck. Be sure to label the axes.

c. Why are the average speed of the car and the average speed of the truck the same?

Problem 3. Blanche and Carlos are traveling to Nashville in separate cars. Carlos stops at a rest stop and waits for Blanche to pass. The graph that follows represents the speeds of their cars in terms of the elapsed time in seconds since Blanche passed the rest stop. Blanche's car is traveling at a constant speed of 62 miles per hour. As she passes the rest stop, Carlos pulls out onto the highway; and they both continue traveling.



a. Which graph represents Blanche's speed and which graph represents Carlos' speed? Explain.

b. Which car is further down the road 20 seconds after being at the rest stop? Explain.

c. Explain the meaning of the intersection point.

d. What is the relationship between the positions of Blanche's car and Carlos' car 27 seconds after passing the rest stop?

e. Carlos catches up with Blanche 64.5 seconds after she passed the rest stop. What is the average speed of Carlos' car over the interval from 0 seconds to 64.5 seconds after leaving the rest stop? Explain.

Problem 4. Let *d* be the distance of a car (measured in feet) from mile marker 420 on a country road and let *t* be the time elapsed (in seconds) since the car passed mile marker 420. The formulas below represent various ways these quantities might be related. For each of the given relationships, determine the average speed of the car using the given formula and the specified time interval.

a. $d = f(t) = t^2$ on the time interval from $t = 5$ to $t = 30$ seconds

b. $d = f(t) = 4.17 \cdot 2.8^t$ on the time interval from $t = 0$ to $t = 3$ seconds

c. $d = f(t) = 3.75\cos(t)$ on the time interval from $t = 10$ to $t = 40$ seconds

Problem 5. For each function f given below, construct the function g_a that gives the average rate of change for the function f on the input interval from $x = a$ to $x = a + h$. Use algebra to simplify your answer as much as possible.

a. $y = f(x) = 12x + 6.5$

b. $y = f(x) = 97$

c. $y = f(x) = 3x^3 - 9$

d. $y = f(x) = 2x^{-1}$

Answers to the Homework.

Problem 1.

Part (a). Both Carrie and Kevin traveled the same distance (5 miles) and they took the same amount of time (17 minutes) to travel those 5 miles.

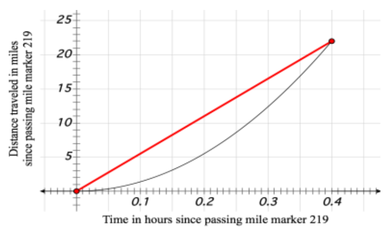
Part (b). Kevin's speed is constant for the entire 17-minute period, Carrie traveled slower in the beginning and gradually sped up.

Part (c). Their average speeds were the same (5/17 miles per minute).

Problem 2.

Part (a). The constant *speed* would be $\frac{|197 - 219|}{\left(24/60\right)} = 55$ miles per hour. (Note that speed does not incorporate direction.)

Part (b). Here is the graph. Your distance versus time graph for the truck could vary.



Part (c). The average speed measures the constant rate at which an object would need to travel in order to produce the same net change in distance in an identical segment of time. Since we only measured the position of the truck and car at the ends of the segment, and they were next to each other at those points and both had traveled the same amount of time, the average speeds are the same.

Problem 3.

Part (a). Since this is a speed versus time graph, a constant speed of 65 mph is represented by a horizontal line at $y = 6.5$. So the horizontal line represents Blanche's speed. Carlos' speed increases from 0 mph at $t = 0$, which is represented by the non-horizontal graph. This graph begins at the origin and the speed of the car increases as the number of seconds since passing the rest stop increases.

Part (b). Twenty seconds after passing the rest stop, Blanche is further down the road because during the entire 20-second interval Blanche was traveling at a speed greater than that of Carlos.

Part (c). The intersection point represents the number of seconds that have elapsed since passing the rest stop when the two cars are traveling the same speed, 62 mph.

Part (d). Twenty-seven seconds after being at the rest stop, the speed of Carlos is just slightly greater than the speed of Blanche. However, this does not imply that Carlos is ahead of Blanche.

Part (e). Since Blanche and Carlos are in the same position 64.5 seconds after both cars past the rest stop, the average speed of Carlos over this interval is the constant speed of Blanche, or 62 miles per hour.

Problem 4.

Part (a). We know that $\Delta t = 25$ seconds, while $\Delta d = f(30) - f(5) = 875$ feet. Thus, the average speed will be

$$\left|\frac{\Delta d}{\Delta t}\right| = 35 \text{ feet per second}$$

Part (b). We know that $\Delta t = 3$ seconds, while $\Delta d = f(3) - f(0) = 29.19$ feet. Thus, the average speed will be

$$\left|\frac{\Delta d}{\Delta t}\right| = 9.73 \text{ feet per second}$$

Part (c). We know that $\Delta t = 30$ seconds, while $\Delta d = f(40) - f(10) \approx 0.6455$ feet. Thus, the average speed will be

$$\left|\frac{\Delta d}{\Delta t}\right| \approx 0.0215 \text{ feet per second}$$

Problem 5.

Part (a). We know that

$$r = g_a(h) = \frac{f(a+h) - f(a)}{h} = \frac{[12(a+h) + 6.5] - [12a + 6.5]}{h} = 12 \quad (h \neq 0)$$

Part(b). We know that

$$r = g_a(h) = \frac{f(a+h) - f(a)}{h} = \frac{[97] - [97]}{h} = 0 \quad (h \neq 0)$$

Part (c). We know that

$$r = g_a(h) = \frac{f(a+h) - f(a)}{h}$$

$$= \frac{[3(a+h)^3 - 9] - [3a^3 - 9]}{h}$$

$$= \frac{[3(a^3 + 3a^2h + 3ah^2 + h^3) - 9] - [3a^3 - 9]}{h} = 9a^2 + 9ah + 3h^2 \qquad (h \neq 0)$$

Part (d). We know that

$$r = g_a(h) = \frac{f(a+h) - f(a)}{h}$$

$$= \left(\frac{1}{h}\right)\left(\frac{2}{a+h} - \frac{2}{a}\right)$$

$$= \left(\frac{1}{h}\right)\left(\frac{2a - 2(a+h)}{a(a+h)}\right) = -\frac{2}{a(a+h)} \qquad (h \neq 0)$$