Whenever we want to make a case to convince an audience that some proposal is true or false, we must base that case on ideas which our audience will accept as true independent of the case we make. The case we make is called an *argument*[1], and the ideas we assume to be true are called *premisses*[2]. The proposal is usually called the *conclusion* of the argument.

Arguments are created and understood by means of a thought process we call *reason*[3] or *inference*[4]. Inference takes place when we affirm the truth or falsehood of a proposal based on the assumed truth of a set of premisses. *Logic*[5] is the branch of philosophy devoted to understanding the methods and principles used to distinguish "correct" reasoning from "incorrect" reasoning. It is widely believed that Aristotle (384-322 BC) first identified the fundamental principles of reasoning.

Ascertainable Sentences and Logical Statements

A *sentence* is the smallest grammatically correct entity that conveys a complete thought. If that thought presents information that has a discernable truth-value at any moment, then the sentence is said to be *ascertainable*. The thought conveyed by an ascertainable sentence is known as a *logical statement*.

An ascertainable[6] sentence is related to a logical statement in the same way that a particular numeric symbol is related to the abstract number it represents. For example, the following ascertainable sentences, while different as sentences, carry the same information content and therefore represent the same logical statement.

- *All humans are mortal.*
- *Every human is mortal.*

Problem 1. Consider the following sentences. Are any of these sentences ascertainable? Explain your thinking.

(a) *Broccoli tastes good.*
(b) *Good Lord, it is past ten o'clock!*
(c) *Don't we know that money talks?*
(d) *Cyclops*[7] *exist.*

[1] The word "argument" comes from the Latin noun "*argumentum*" which means "the instrument by which I demonstrate."
[2] The word "premiss" as it is used here comes from the Latin verb "*praemittere*" which means "to send or put before."
[3] The word "reason" comes from the Latin verb "*rerī*" which means "to think."
[4] The word "infer" comes from the Latin verb "*inferre*" which means "to bring into."
[5] The work "logic" comes from the Greek verb "*logizesthai*" which means "to calculate" or "to reckon."
[6] The word "ascertain" comes from the French verb "*acerteiner*" which means "to assure."
[7] The cyclops is a creature from Greek mythology.

Argument

An *argument* is a collection of at least two logical statements (which may be compounded into fewer sentences). All but one of these statements are presented as grounds for believing the truth of the remaining statement. The supporting statements are known as *premisses*, and the statement they are presented to support is called the *conclusion* of the argument.

Problem 2. Do any of the following sentences represent arguments? Explain your thinking.

Part (a). If it rains today, then I will go to the store.

Part (b). Objects of art are expensive because objects of art are a language unto themselves.

Part (c). Synonyms are good servants but bad masters; therefore select them with care[8].

Problem 3. Which, if any, of the following arguments do you think represents "correct" reasoning? Explain your thinking.

Argument 1
Premiss: "On a clear day, the sky is blue."
Premiss: "Today is Tuesday."
Consequently…
Conclusion: "I need to go to Slick Pig today."

Argument 2
Premiss: "Every Tuesday, Slick Pig has its barbecue on sale for half price."
Premiss: "Today is Tuesday."
Consequently …
Conclusion: "I need to go to Slick Pig today."

[8] Taken from the preface of Roget's *Thesuarus*.

Problem 4. Consider the argument below. This argument is taken from "Carter and Women," an article by Mim Kelber, that appeared in the June 14, 1980 edition of *The Nation*.

Part (a). Read this argument and list its conclusion and the premisses used to support it.

Part (b). Do you think this argument represents "correct" reasoning? Explain.

When I voted for Jimmy Carter in 1976, I, too, thought he showed a sensitivity to the needs of working women. I can no longer think so. He has run out on his commitment to a comprehensive child-care program, a central and essential need of working parents. Continuing the slashes in his 1980 budget, the 1981 budget eliminates another 50,000 Comprehensive Employment and Training Act jobs, depriving chronically unemployed and poor women of work opportunities. His decontrol of oil prices worsened inflation, a particular burden on working women, who earn on average only 59 percent of what men earn. His deliberate creation of a recession has produced mass unemployment, with women showing higher percentages of joblessness than men and forcing men and women to compete with one another for increasingly scarce jobs.[9]

As you can see, it is not always easy to identify the component parts of an argument. In mathematics, we usually try to make the structure of an argument more clear by utilizing certain key words or phrases to identify premisses and conclusions. Here are some commonly used key phrases.

Premiss Indicators		Conclusion Indicators	
• Since	• As indicated by	• Therefore	• We may infer
• Because	• The reason is that	• Consequently	• I conclude that
• For	• For the reason that	• Hence	• Which shows that
• We know that	• May be inferred from	• Thus	• As a result
• Follows from	• May be deduced from	• It follows that	• Which implies that
• As shown by	• In view of the fact that	• As shown by	• Points to the conclusion
• Inasmuch as	• To deny this would be	• So	• Which allows us to infer
• Otherwise	• May be derived from	• Accordingly	• Ergo

Problem 5. Consider the Supreme Court decision handed down in the 1952 case Zorach versus Clauson[10]. Recast this argument, making the conclusion and premisses clear using indicators.

It takes obtuse reasoning to inject any issue of the "free exercise" of religion into the present case. No one is forced to go to the religious classroom and no religious exercise or instruction is brought to the classrooms of the public schools. A student need not take religious instruction. He is left to his own desires as to the manner or time of his religious devotions, if any.

[9] This excerpt appeared in Introduction to Logic, 6th Edition, by Irving Copi, MacMillan publishing company, 1983, page 8.
[10] *Ibid*., page 10.

Arguments, whether representing "correct" or "incorrect" reasoning, are classified in two broad types. An argument falls into a particular type based on what its author claims the premisses accomplish.

Argument Types

- An argument is *inductive* if its premisses are presented to suggest *probable cause* for the truth of a conclusion. An inductive argument is classified from *very strong* to *very weak* based on the degree to which it accomplishes its goal. Even in a very strong inductive argument the truth of the premisses *does not guarantee* the truth of the proposal.
- An argument is *deductive* if its premisses are presented to provide *definitive proof* that the conclusion is true. If this goal is accomplished, the argument is said to be *valid*; if not, it is said to be *invalid*. In a valid deductive argument, the truth of the premisses *guarantees* the truth of the conclusion. An invalid deductive argument is called a *fallacy*.

Henceforth, we will dispense with the terms "correct" and "incorrect" when referring to the reasoning behind arguments. The argument appearing in Problem 4 above is inductive[11]; this is the case for the majority of arguments you encounter in everyday life.

It is often difficult to classify a particular argument, however; there is a loose test that can be applied: A deductive[12] argument remains valid[13] even if additional premisses are added to the argument. Adding premisses to an inductive argument may strengthen or weaken the argument or change the truth-value of its conclusion altogether.

Problem 6. Classify the arguments presented below. How did you decide on your classification?

Argument 1
Premiss: "All dogs are mammals."
Premiss: "My pet Freddy is a dog."
Consequently…
Conclusion: "My pet Freddy is a mammal."

Argument 2
Premiss: "We know that $8 = 2^3$, and $8/2 = 2^2$."
Premiss: "We know that $4 = 2^2$, and $4/2 = 2^1$."
Premiss: "We know that $2 = 2^1$, and $2/2 = 1$."
Consequently …
Conclusion: "It makes sense to define the power 2^0 to be the number 1."

Problem 7. Consider the following argument.
Part (a). Would you classify this as an inductive or a deductive argument? Explain your thinking.

Part (b). If you think the argument is deductive, is it valid or invalid? If you think the argument is inductive, how would you classify this argument on the very strong to very weak scale? Explain your thinking.

Premiss: "Twelve months ago, Jared weighed 120 pounds."
Premiss: "Jared ate a teaspoon of yeast every day for the past year."
Premiss: "Jared weighed 155 pounds yesterday."
Consequently,
Conclusion: "Eating yeast causes people to gain weight."

[11] The word "inductive" arises from the Latin verb "*inducere*" which means "to persuade."
[12] The word "deductive" arises from the Latin verb "*deducere*" which means "to lead down."
[13] The word "valid" comes from the Latin verb "*valere*" which means "to be strong."

Problem 8. Consider the following argument.
Part (a). Would you classify this as an inductive or a deductive argument? Explain your thinking.

Part (b). If you think the argument is deductive, is it valid or invalid? If you think the argument is inductive, how would you classify this argument on the very strong to very weak scale? Explain your thinking.

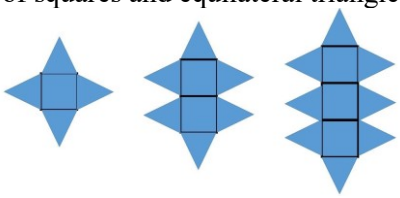
Premiss: "If Donald Trump owned Fort Knox, then he would be wealthy."
Premiss: "Donald Trump does not own Fort Knox."
Consequently,
Conclusion: "Donald Trump is not wealthy."

Problem 9. Consider the following argument.
Part (a). Would you classify this as an inductive or a deductive argument? Explain your thinking.

Part (b). If you think the argument is deductive, is it valid or invalid? If you think the argument is inductive, how would you classify this argument on the very strong to very weak scale? Explain your thinking.

Premiss: "Every warm-blooded animal has wings."
Premiss: "All birds are warm-blooded."
Consequently,
Conclusion: "All birds have wings."

Problem 10. The first three figures in a long sequence of geometric figures are shown below. Each figure in the sequence consists only of squares and equilateral triangles.



How many triangles will likely appear in the tenth figure of this sequence?

Part (a). Write down your conclusion.

Part (b). Write down an argument to support your conclusion. Is your argument inductive or deductive?