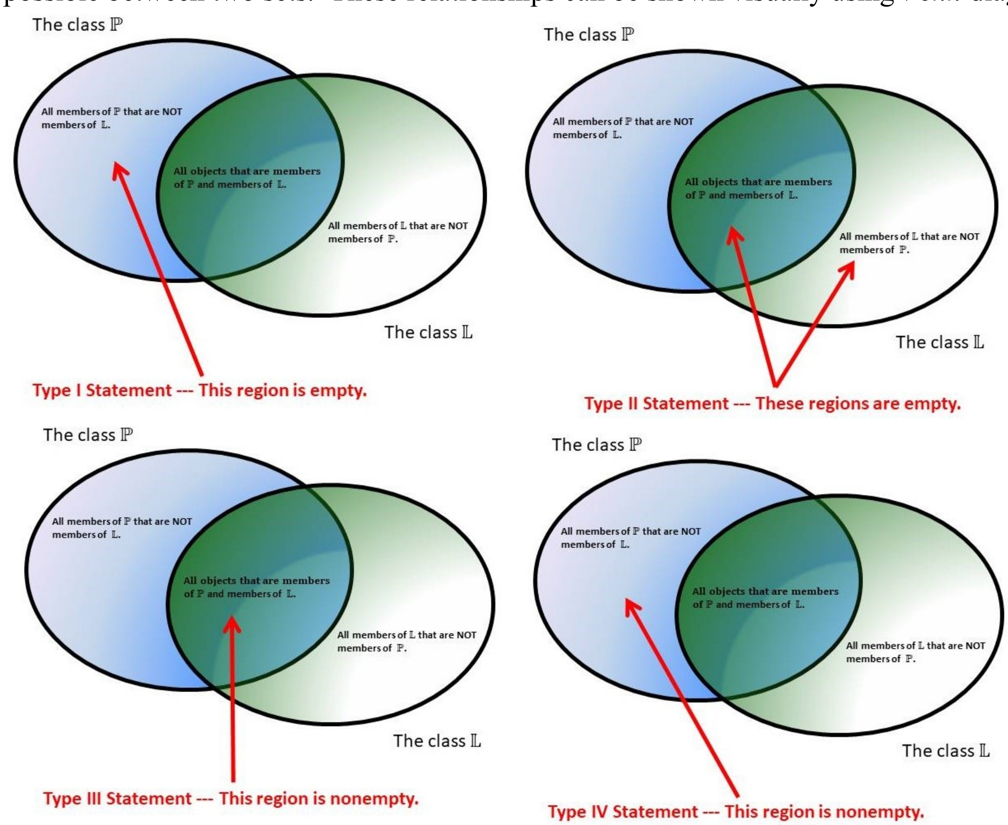
As mentioned in Investigation 1, a deductive argument is one whose premisses, when true, are expected to guarantee the truth of its conclusion. The theory of deduction explains the relationship between the premisses and conclusion of a valid argument and provides techniques for determining which deductive arguments are valid.

The theory of deduction seems to have originated with the Greek philosopher Aristotle. His approach focused on arguments constructed from *categorical* statements, and that has essentially remained the case to this day.

In the theory of deductive logic, a family of members that have some particular characteristic in common is called a *class*[1]. A logical statement is *categorical* when it describes the way two classes are related. Aristotle identified four types of categorical statements. To illustrate these types, let's consider the class $\mathbb{P}$ of all politicians and the class $\mathbb{L}$ of all liars.

- Type I (Universal Affirmative) --- "*All politicians are liars*."
- Type II (Universal Negative) --- "*No politician is a liar.*"
- Type III (Existential Affirmative) --- "*At least one politician is a liar.*"[2]
- Type IV (Existential Negative) --- "*At least one politician is not a liar.*"

In modern terminology, we would think of these statements as referring to the various inclusion relations possible between two sets. These relationships can be shown visually using *Venn* diagrams.[3]



[1] We might be tempted to refer to such collections as *sets* … but that is a story for later.
[2] Some logicians prefer the word "some," but this word is seldom used for mathematical discourse.
[3] Venn diagrams were introduced by John Venn in the 1880's and provide a convenient way to visualize the relationships between members of two or three sets.

Ascertainable sentences will always make an assertion concerning the inclusion of members from one class (the *subject class*) as members of another class (the *object class*). The *quality* of the underlying logical statement is determined by whether this assertion is affirmative or negative.

- The statement has *affirmative quality* provided it asserts that all or some members of the subject class are members of the object class.
- The statement has *negative quality* provided it asserts that all or some members of the subject class are NOT members of the object class.

The subject class is usually[4] introduced with a word or phrase synonymous with one of "*all*," "*no*," or "*there exist*." These lead words are called *quantifiers*, and they determine the *quantity* of the categorical statement.

- Ascertainable sentences whose subject class is introduced by "all" or "no" represent logical statements that are *universal* in quantity.
- Ascertainable sentences whose subject class is introduced by "there exist" represent logical statements that are *existential* in quantity.

Problem 1. The following ascertainable sentences represent categorical statements that are *in opposition.* Based on these pairs, how would you define this term?

- "*All dogs are mammals.*"
- "*At least one dog is a mammal.*"

- "*There exist coats that are used in cold weather.*"
- "*There exist coats that are not used in cold weather.*"

- "*All musicians are artists.*"
- "*There exist musicians who are not artists.*"

Problem 2. The following ascertainable sentences represent categorical statements. Are these statements in opposition? Explain.

- "*All dogs are mammals.*"
- "*There exist dogs that are not four-legged animals.*"

Negation of a Logical Statement
The *negation* of a logical statement is a new logical statement that is FALSE precisely when the original statement is TRUE.

[4] Sentences representing categorical statements can be rephrased so this is always the case.

Creating an ascertainable sentence that represents a logical statement which is the negation of another logical statement is done using the typical rules of grammar. For example, consider the ascertainable sentence

"*I went to the store this morning.*"

Two ascertainable sentences that represent the negation of the underlying logical statement are

- "*I did not go to the store this morning.*"
- "*It is not the case that I went to the store this morning.*"

Problem 3. Consider the categorical statement represented by the ascertainable sentence "All politicians are liars." Is the negation of this statement represented by one of the three ascertainable sentences below? Use the Venn diagrams to justify your answer.

- "*No politician is a liar.*"
- "*At least one politician is a liar.*"
- "*At least one politician is not a liar.*"

Problem 4. What would be an ascertainable sentence that represents the negation of the logical statement represented by "*No politicians are liars*"? Use the Venn diagrams above to justify your claim.

Problem 5. Two categorical statements are *contradictory* provided one is the negation of the other. Based on your work in the previous two problems, what ascertainable sentence would represent the contradictory for the logical statement represented by

"*At least one ruby is not a precious stone.*"

Problem 6. Two categorical statements are *contraries* provided they make assertions about the same subject and object classes that cannot both be TRUE but could both be FALSE. Provide an example of two ascertainable sentences representing statements that are contraries but are *not* contradictory.

Problem 7. Consider the following ascertainable sentence.

"*All squares are rectangles.*"

Part (a). Is there a statement contradictory to the underlying statement represented by this sentence? Explain. Construct an ascertainable sentence that represents this statement if you believe it exists.

Part (b). Is there a statement contrary to the underlying statement represented by this sentence? Explain. Construct an ascertainable sentence that represents this statement if you believe it exists.

Problem 8. Consider the following ascertainable sentences.

- "*There exist crystals that are precious stones.*"
- "*There exist crystals that are not precious stones.*"

The underlying logical statements represented by these sentences are called *sub-contraries*. Based on this example, how would you define sub-contrary statements?

Exercises

Problem 1. Each of the ascertainable sentences below represents a categorical statement. Identify the subject and object class of each statement. (Use the sentence "The subject class is the family of all … and the object class is the family of all …" .) Also, identify the quality and quantity of each statement.

Part (a). "*All new labor-saving devices are not a threat to the labor movement.*"

Part (b). "*There exist people who are not artists that are responsible critics of our project.*"

Part (c). "*Amateurs include no athletes who have received pay for play*."

Problem 2. Construct an ascertainable sentence that represents the negation of the categorical statements represented by the following sentences. Be careful to check your answer. Venn diagrams help.

Part (a). "*There exist ellipses that are circles*."

Part (b). "*No amateur competitor is not a good sport.*"

Part (c). "*There exists no function that is not differentiable.*" (It helps to rephrase this sentence.)

Problem 3. Consider the following pairs of ascertainable sentences. Decide whether these pairs represent contrary statements, sub-contrary statements, or neither.

Part (a). "*There exist adult humans that are one meter tall.*"
"*There exist adult humans that are not one meter tall.*"

Part (b). "*There exist members of this audience who bought tickets on Tuesday.*"
"*All members of this audience bought tickets on Tuesday.*"

Part (c). "*All chain link fences are objects that contain metal parts*."
"*All chain link fences are not objects that contain metal parts*."