There are two types of deductive reasoning. *Immediate inference* occurs when the conclusion of an argument is deduced from a single premiss. *Mediate[1] inference* occurs when the conclusion of an argument is deduced from one premiss through the mediation of other premisses. The ancient Greeks determined that valid immediate inference can involve only categorical statements.

Problem 1. Consider the pairs of ascertainable sentences below. In each case, the pairs of underlying statements are said to be *converses*. Based on these pairs, how would you characterize two categorical statements that are converses?

- "*All tea bags are things made of paper.*"
- "*All things made of paper are tea bags.*"

- "*There exist continuous functions that are not differentiable functions.*"
- "*There exist not differentiable functions that are continuous functions.*"

- "*No flower that blooms at night is a toxic plant.*"
- "*No toxic plant is a flower that blooms at night.*"

Problem 2. In logic, the *complement* of a class is the family of all things that are *not* members of the class[2]. Consider the class of all red flowers. Would the complement of this class be the class of all flowers that are *not* red? Explain.

Problem 3. Consider the pairs of ascertainable sentences below. In each case, the pairs of underlying statements are said to be *obverses[3]*. Based on these pairs, how would you characterize two statements that are obverses?

- "*All Chinese immigrants are illegal aliens.*"
- "*No Chinese immigrants are non-(illegal aliens)[4].*"

- "*There exist carnations that are red flowers.*"
- "*There exist carnations that are not non-(red flowers).*"

- "*No piecewise-defined function is a function that is continuous on its domain.*"
- "*All piecewise-defined functions are non-(functions continuous on their domain) .*"

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[1] In this context, the word "mediate" is an adjective that means "in between." It is pronounced the same as in "immediate;" which, of course, means "not in between."

[2] It is customary to attach the word "non" to the verbal description of a class in order to describe it's complement.

[3] The word "obverse" comes from the Latin word "*obversus*" which means "turned against" or "facing."

[4] To avoid confusion, we will often use parentheses in the verbal description of a class complement.

Problem 4. The *contrapositive* of a categorical statement is the categorical statement whose subject class is the complement of the object class in the original statement and whose object class is the complement of the subject class in the original statement. The quality and quantity of both statements is the same. Based on this definition, construct an ascertainable sentence representing the contrapositive of the statements represented by the following statements.

Part (a). "*There exist non-(roller coasters) that are metallic objects.*"

Part (b). "*All prime integers are non-(composite integers).*"

Problem 5. Consider the categorical statement represented by the ascertainable sentence below.
                                        "*No dog is a cat.*"
Construct ascertainable sentences that represent the obverse, contrapositive, and converse of this statement.

- Converse ---
- Contrapositive ---
- Obverse ---

Problem 6. Consider the sentences you constructed in Problem 5. If we assume the sentence "*No dog is a cat*" represents a true statement, is the converse, contrapositive, or obverse also a true statement? Explain.

Problem 7. Consider the categorical statement represented by the ascertainable sentence below.
                                "*All snakes are cold-blooded animals.*"
Construct ascertainable sentences that represent the obverse, contrapositive, and converse of this statement.

- Converse ---
- Contrapositive ---
- Obverse ---

Problem 8. Consider the sentences you constructed in Problem 7. If we assume the sentence "*All snakes are cold-blooded animals*" represents a true statement, is the converse, contrapositive, or obverse also a true statement? Explain.

Problem 9. Consider the affirmative categorical statement represented by the ascertainable sentence below.

"*There exist citizens who are non-legislators.*"

Construct ascertainable sentences that represent the obverse, contrapositive, and converse of this statement.

- Converse ---
- Contrapositive ---
- Obverse ---

Problem 10. Consider the sentences you constructed in Problem 9. If we assume the sentence "*There exist citizens who are non-legislators*" represents a true statement, is the converse, contrapositive, or obverse also a true statement? Explain.

Problem 11. Consider the negative categorical statement represented by the ascertainable sentence below.

"*There exist paid professionals who are not athletes.*"

Construct ascertainable sentences that represent the obverse, contrapositive, and converse of this statement.

- Converse ---
- Contrapositive ---
- Obverse ---

Problem 12. Consider the sentences you constructed in Problem 11. If we assume the sentence "*There exist paid professionals who are not athletes*" represents a true statement, is the converse, contrapositive, or obverse also a true statement? Explain.

The Law of Obversion, Conversion, and Contraposition

- Given any categorical statement, we may immediately deduce its obverse.
- Given any universal negative or existential affirmative statement, we may immediately deduce its converse.
- Given any universal affirmative or existential negative statement, we may immediately deduce its contrapositive.

Any other immediate inference drawn from a categorical statement is not valid, as the resulting conclusion can be false when the original categorical sentence is true.

Problem 13. Is the following argument valid? Explain.

There exist tabby cats that are animals with twenty-one toes. Therefore, there exist tabby cats that are not animals with twenty-one toes.

Problem 14. Is the following argument valid? Explain.

All women are people who are created equal. Therefore, all people who are created equal are women.

Problem 15. Is the following argument valid? Explain.

There exist men who are not women. Therefore, there exist non-women who are men.

Problem 16. Is the following argument valid? Explain.

There exist men who are not women. Therefore, there exist women who are not men.

Problem 17. Is the following argument valid? Explain.

No men are women. Therefore, all men are non-women.

Exercises

Problem 1. Consider the class $\mathbb{P}$ of all people who are friendly. Does the class of all things that are *not* members of the class non-$\mathbb{P}$ contain members of $\mathbb{P}$? Are these classes the same? Explain.

Problem 2. Write a sentence that represents the converse, obverse, and contrapositive of the statements represented by the following sentences.

Part (a). "*No pigs are animals that can fly.*"
- Converse ---
- Contrapositive ---
- Obverse ---

Part (b). "*There exist animals that can fly which are not pigs.*"
- Converse ---
- Contrapositive ---
- Obverse ---

Part (c). "*All things that rise are things that must converge.*"
- Converse ---
- Contrapositive ---
- Obverse ---

Problem 3. Assume the following sentence represents a true statement.

"*There exist mathematicians who are not country singers.*"

Based on this assumption only, which of the following sentences represent statements that are necessarily true, necessarily false, or impossible to determine? Justify your answer.

Part (a). "*There exist country singers who are not mathematicians.*"

Part (b). "*There exist non-(country singers) who are not non-mathematicians.*"

Part (c). "*There exist non-(country singers) who are non-mathematicians.*"

Part (d). "*All mathematicians are country singers.*"

Part (e). "*All non-(country singers) are non-mathematicians.*"

Part (f). "*No country singers are mathematicians.*"

Part (g). "*All mathematicians are not non-(country singers).*"

Part (h). "*There exist non-(country singers) who are mathematicians.*"

Problem 4. Is the following argument valid? Explain.

No man is an island. Therefore, no island is a man.

Problem 5. Is the following argument valid? Explain.

All my ex's are women who live in Texas. Therefore, all the women who live in Texas are my ex's.

Problem 6. Is the following argument valid? Explain.

All the gold in California is something in a vault in Beverly Hills. Therefore, all the gold in California is not something which isn't in a vault in Beverly Hills.

Problem 7. Is the following argument valid? Explain.

There exists a rose which is something in Spanish Harlem. Therefore, there exists something in Spanish Harlem which is a rose.

Problem 8. Is the following argument valid? Explain.

There exist parts of the pine tree which are not edible things. Therefore, there exist inedible things which are not parts of the pine tree.