In the previous investigation, we explored valid and invalid ways to make immediate inferences from single categorical statements.   In this investigation, we take the process of deduction a step further.

Problem 1. Consider the following ascertainable sentences.

- *"All odd integers divisible by 2 are prime numbers."*
- *"No odd integer divisible by 2 is a prime number."*

Part (a). Are the underlying statements contradictory? Explain.

Part (b). Are the underlying statements contrary? Explain.

Part (c). Are the underlying statements sub-contrary? Explain.

Existential Import

A categorical statement is said to have *existential import* provided the truth of the statement guarantees its subject class is nonempty.

Problem 2. Consider the ascertainable sentences below.

- *"All politicians are liars."*
- *"No politician is a liar."*

Do either of the underlying statements have existential import? Explain your thinking.

Problem 3. Construct an ascertainable sentence of your own representing a statement that has existential import.

Problem 4. Consider any statement with existential import. Is it *necessary* for this statement to be true in order for its subject class to be nonempty? Justify your answer.

> **Statement Quantity and Existential Import**
>
> A categorical statement having universal quantity *does not* have existential import. Categorical statements having existential quantity do have existential import. Statement quality does not affect existential import.

Problem 5. Consider the following arguments. Do you think these arguments are valid? Explain.

- "*All animals that have gills are fish.*"
- "*There exist animals that have gills which are fish.*"

- "*No leprechauns live on my property.*"
- "*Consequently, there exist leprechauns that do not live on my property.*"

> **The Fallacy of Existential Import**
>
> You *may not* validly deduce an existential statement immediately from a universal statement. Such a deduction also requires the premiss that the subject class of the universal statement is nonempty.

The following arguments are examples of *transitive reasoning.*

• There exist birds that are warm-blooded animals. • There exist warm-blooded animals that have two legs. • Therefore, there exist birds that have two legs.	• There exist bolts that are not screws. • There exist screws that are not nuts. • Therefore, there exist bolts that are not nuts.	• No fish are animals that have legs. • No animal that has legs breathes methane. • Therefore, no fish breathes methane.

**Problem 7.** Based on the structure of these arguments, how would you define transitive reasoning?

Problem 8. Do you think any of these arguments are *valid*? Justify your answer. (Venn diagrams can help organize your thoughts.)

Problem 9. Let $\mathbb{P}$, $\mathbb{Q}$, and $\mathbb{R}$ represent classes of objects.

Part (a). Assuming the statements represented by the sentences below are TRUE, shade in the region on the Venn diagram that corresponds to each.
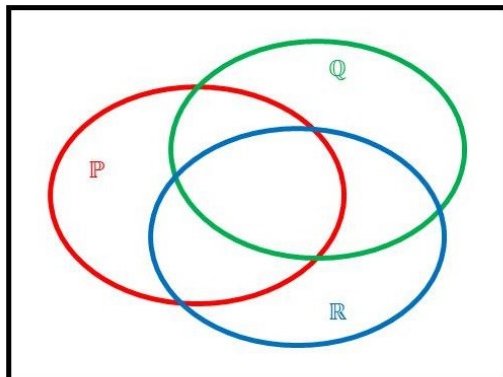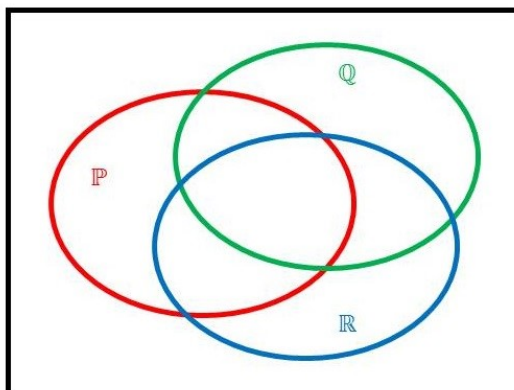
All members of $\mathbb{P}$ are members of $\mathbb{Q}$.
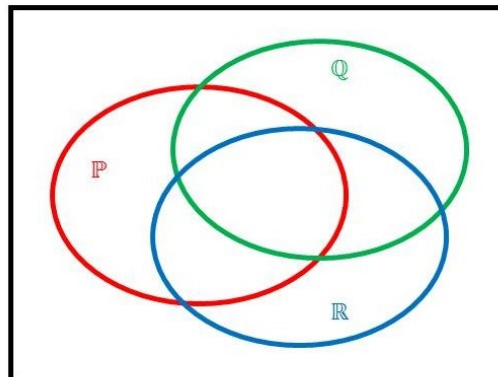
All members of $\mathbb{Q}$ are members of $\mathbb{R}$.

Part (b). Assume both statements are simultaneously TRUE and shade in the region on the Venn diagram that corresponds to this situation.

Part (c). Looking at the shaded region in Part (b), what categorical statement is *guaranteed* to be TRUE?

Valid Transitive Categorical Reasoning

Transitive categorical reasoning is valid when applied to universal affirmative statements.

Problem 10. Analyze the following argument by identifying laws or techniques used to reach the final conclusion. Is this argument valid? Explain.

- *All elected officials are citizens of this country.*
- *All citizens of this country are taxpayers.*
- *Therefore, all elected officials are taxpayers.*
- *Consequently, all non-taxpayers are non-elected officials.*
- *Hence, there exist non-taxpayers who are non-elected officials.*
- *We may therefore conclude that there exist non-elected officials who are non-taxpayers.*

Exercises

We say that two statements are *materially equivalent* provided they always have the same truth value.

Problem 1. Let $\mathbb{P}$ and $\mathbb{Q}$ represent classes of objects. Do the following pairs of ascertainable sentences represent materially equivalent statements? Justify your answer. (Venn diagrams can be helpful.)

First Pair
- *No member of the class $\mathbb{P}$ is a member of the class $\mathbb{Q}$.*
- *No member of the class non-$\mathbb{Q}$ is a member of the class $\mathbb{P}$.*

Second Pair
- *No member of the class $\mathbb{P}$ is a member of the class $\mathbb{Q}$.*
- *All members of the class $\mathbb{P}$ are members of the class non-$\mathbb{Q}$.*

Problem 2. There are many variations on transitive categorical reasoning … some valid and some not. Is the following variant on transitive reasoning valid? Explain.

- *No animal that flies is a thing that can swim.*
- *All things that cannot swim have beady eyes*.
- *Therefore, all animals that fly have beady eyes*.

Problem 3. Is the following argument valid? Justify your answer.

- *There exist great scientists.*
- *All great scientists are college graduates.*
- *There exist college graduates who are professional athletes.*
- *Therefore, there exist great scientists who are professional athletes.*

Problem 4. Is the following argument valid? Justify your answer.

- *There exist mathematicians who are not vegetarian.*
- *There exist journalists who are vegetarian.*
- *Therefore, there exist journalists who are not mathematicians.*

Problem 5. Is the following argument valid? Justify your answer.

- *All vegetarians are economists.*
- *No economists are non-mathematicians.*
- *Therefore, all vegetarians are mathematicians.*

Problem 6. Analyze the following argument, indicating which laws or techniques are used to reach the final conclusion. (You may need to add some mediate premisses.) Is this argument valid? Justify your answer.

- *Anna is a member of Miss Trixie's dance troupe.*
- *All members of Miss Trixie's dance troupe are kleptomaniacs[1].*
- *All kleptomaniacs are people in need of psychotherapy.*
- *Therefore, there exist people who do not need psychotherapy who are not members of Miss Trixie's dance troupe.*

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[1] A kleptomaniac is a person with an uncontrollable urge to steal.