Since many ascertainable sentences can represent the same logical statement, it makes sense to avoid the confusion of verbal clutter and use symbols to denote logical statements when we want to manipulate them. It is customary to use lowercase roman letters to denote logical statements.

For example, we can write "Let p denote the logical statement represented by all ascertainable sentences presenting the same thought as the sentence *My dog has fleas*." Of course, we usually shorten this description simply to read "Let p denote the logical statement represented by the sentence "*My dog has fleas*."

Problem 1. Consider the following ascertainable sentence.

- "*The man who shot President Lincoln was an actor.*"

Part (a). There is one *sub-sentence*[1] of this sentence which is also ascertainable. What is it?

Part (b). If you replace this sub-sentence in the original sentence with *any* ascertainable sentence you like, will the resulting sentence always be ascertainable? Explain your thinking.

Problem 2. Consider the following ascertainable sentence.

- "*If I get paid this morning, then I will buy you a fruit cup this afternoon.*"

Part (a). There are two *sub-sentences* of this sentence which are also ascertainable. What are they?

Part (b). Pick one of your ascertainable sub-sentences. If you replace this sub-sentence in the original sentence with *any* ascertainable sentence you like, will the resulting sentence always be ascertainable? Explain your thinking.

Components of a Statement

Suppose that p and q denote logical statements. We say that q is a *component* of p provided the following conditions are met

- Any ascertainable sentence representing p contains an ascertainable sub-sentence representing q.
- We can replace the ascertainable sub-sentence representing q with an ascertainable sentence representing any other statement, and the result will be a new ascertainable sentence.

[1] A *sub-sentence* of a sentence is a collection of consecutive words taken from the original sentence which also forms a sentence.

We say that a logical statement is *simple* if it contains no component statements. A logical statement which does contain component statements is said to be *compound*.

Problem 3. Consider the following compound statement. Let p denote the statement represented by this sentence.

- "*Barney believes that Fred loves Wilma*."

Let q denote the component statement represented by the sub-sentence "*Fred loves Wilma*." Do you think that the truth-value of p *depends* on the truth-value of q? Explain.

We say that a logical statement is *truth-functional* provided its truth-value is completely determined by the truth-value of some collection of logical statements all different from the statement itself. We can always create a truth-functional statement from a collection of logical statements by explaining precisely how its truth value is determined by the truth-values of the collection. Here are two useful examples of compound truth-functional statements.

Logical Disjunctive and Conjunctive Statements

Suppose that p and q denote logical statements.

- The *logical disjunction* is the statement that is FALSE precisely when both p and q are FALSE. This statement is denoted by the symbol $p \vee q$.
- The *logical conjunction* is the statement that is TRUE precisely when both p and q are TRUE. This statement is denoted by the symbol $p \wedge q$.

If p denotes the logical statement represented by the sentence "*My dog has fleas*," and q denotes the logical statement represented by the sentence "*Today is Tuesday*," then we have the following grammatical interpretation for the logical disjunction and conjunction.

- We interpret $p \vee q$ to mean "*My dog has fleas, OR today is Tuesday.*"[2]
- We interpret $p \wedge q$ to mean "*My dog has fleas, AND today is Tuesday.*"

The symbols $\vee$ and $\wedge$ are examples of logical *connectives*. Truth-functional compound statements are usually constructed using logical connectives.

There are a number of English words which can be used in place of the connective "and." For example,

- "*My dog has fleas, AND today is Tuesday*" conveys the same thought as "*My dog has fleas, BUT today is Tuesday*."
- "*My dog has fleas, AND today is Tuesday*" conveys the same thought as "*My dog has fleas, MOREOVER today is Tuesday*."
- "*My dog has fleas, AND today is Tuesday*" conveys the same thought as "*My dog has fleas, ALSO today is Tuesday*."
- "*My dog has fleas, AND today is Tuesday*" conveys the same thought as "*My dog has fleas, HOWEVER today is Tuesday*."

[2] The special symbol "$\vee$" originates from the Latin connective "*vel*" which means "or" in the inclusive sense.

There is built-in ambiguity in the English word "or." English speakers use the word "or" in two senses --- An *inclusive* sense in which the compound statement will be true when *one or both* component statements are true and an *exclusive* sense in which the compound statement will be true when *only one* of the component statements is true. The following ascertainable sentences provide examples.

- Inclusive sense --- "*I washed my car today, OR Joan went to the store.*"
- Exclusive sense --- "*You get a discount on your meal when you order the appetizer or you order the salad.*"[3]

Problem 4. Consider the ascertainable sentence "*Today is Tuesday, OR today is Wednesday.*" Do you think the "or" is being used in the inclusive or the exclusive sense? Explain your thinking.

A disjunctive statement utilizes "or" in the inclusive sense. The phrase "Either … or" can be used in place of the connective "or" in a disjunctive statement. There are no other common synonyms for the connective "or" used in logic. For example,

- "*My dog has fleas, OR today is Tuesday*" conveys the same thought as "*EITHER my dog has fleas, OR today is Tuesday*."

Problem 5. Recast the following ascertainable sentence as a statement in symbolic language, using parentheses when appropriate.

"*Either Saudi Arabia buys more warplanes and Iran raises the price of oil, or Jordan requests more American aid*."

A truth-functional compound statements can be defined by constructing a *truth table*. A truth table shows the truth value of a compound statement corresponding to the truth values of its component statements.

Truth Table for the Disjunction

p	q	$p \vee q$
TRUE	TRUE	TRUE
FALSE	TRUE	TRUE
TRUE	FALSE	TRUE
FALSE	FALSE	FALSE

Problem 6. Use the definition of the conjunction to complete the following truth table.

Truth Table for the Conjunction

p	q	$p \wedge q$
TRUE	TRUE	
FALSE	TRUE	
TRUE	FALSE	
FALSE	FALSE	

[3] It is usually difficult to determine whether "or" appearing in a sentence is inclusive or exclusive; we often add extra phrasing to clarify.

Problem 7. Consider the definition of the conjunctive statement. Do you think the statement $p \wedge q$ is materially equivalent to the statement $q \wedge p$? Explain your thinking.

Problem 8. Consider the following ascertainable sentences.

- "*Anna took off her shoes, and then Anna got into bed.*"
- "*Anna got into bed, and then Anna took off her shoes*."

Part (a) Do you think the logical statements represented by these sentences are materially equivalent? Explain your thinking.

Part (b). Do you think either of the logical statements represented by these sentences is a conjunction? Explain your thinking.

Problem 9. Let p and q be logical statements.

Part (a). Complete the truth table below for the compound statement $p \wedge (p \vee q)$.

p	q	$p \vee q$	$p \wedge (p \vee q)$.
TRUE	TRUE		
FALSE	TRUE		
TRUE	FALSE		
FALSE	FALSE		

Part (b). Do you think the statement $p \wedge (p \vee q)$ is materially equivalent to the statement p? Explain.

A truth-functional statement does *not* have to be a compound statement. Given any logical statement p, then *negation* of p is defined to be the logical statement that is TRUE precisely when p is FALSE. This truth-functional logical statement, which is simple whenever p is simple, will be denoted by the symbol $\sim p$. We read this symbol as "the negation of p" or simply "not p."

Problem 10. Let p and q be logical statements. Complete the following truth tables and explain why these tables show the logical statements $\sim(p \wedge q)$ and $\sim p \vee \sim q$ are materially equivalent.

p	q	$p \wedge q$	$\sim(p \wedge q)$
TRUE	TRUE		
FALSE	TRUE		
TRUE	FALSE		
FALSE	FALSE		

p	q	$\sim p$	$\sim q$	$\sim p \vee \sim q$
TRUE	TRUE			
FALSE	TRUE			
TRUE	FALSE			
FALSE	FALSE			

Problem 11. Let p and q be logical statements. The *exclusive or* is defined to be the logical statement that is FALSE precisely when both p and q have the same truth-value. We denote this statement using the symbol $p \oplus q$.

Part (a). Construct the truth table for $p \oplus q$.

Part (b). Use conjunctions, disjunctions, and/or negations to construct a symbolic expression that is materially equivalent to $p \oplus q$.

Exercises

Let p, q, and r denote logical statements. Use truth tables to establish the following.

Problem 1. Show that p is materially equivalent to $\sim(\sim p)$. [The Law of Double Negation]

Problem 2. Show that $\sim(p \vee q)$ is materially equivalent to $\sim p \wedge \sim q$. [DeMorgan's Law]

Problem 3. Show that $p \vee p$ is materially equivalent to p. [The Idempotent Law]

Problem 4. Show that $p \vee (q \wedge r)$ is materially equivalent to $(p \vee q) \wedge (p \vee r)$. [Distributive Law]

In the following problems, let

- p denote the logical statement represented by the sentence "*Gilligan loves Mary Ann*."
- q denote the logical statement represented by the sentence "*Ginger hates Squiggy*."
- r denote the logical statement represented by the sentence "*Shirley is not from Ork*."

Problem 5. Construct a symbolic expression to denote the logical statement represented by each of the following ascertainable sentences. Use parentheses when appropriate.

Part (a). *Ginger hates squiggy; moreover, Shirley is from Ork.*

Part (b). *Gilligan loves Mary Ann and Shirley is not from Ork, or Ginger does not hate squiggy.*

Part (c). *Gilligan does not love Mary Ann; also, either Ginger hates Squiggy or Shirley is from Ork.*

Problem 6. Construct an ascertainable sentence that represents the compound statement. Use the appropriate contradiction statements in the negation.

Part (a). *All horned toads are lizards, and there exist lizards which are not horned toads.*

Part (b). *There exist animals that live on other worlds, or all animals live on Planet Earth.*

Part (b). *There exist people who are fooled some of the time, but no people are fooled all of the time.*