

The symbolic approach to manipulating logical statements was introduced by George Boole in the mid 1800's. This approach has come to be known as the *propositional calculus*; it has allowed logicians to generalize and extend much of the logic formulated by Aristotle.

A *syllogism*<sup>1</sup> is a particular deductive argument type that you encountered in Investigation 4. A (categorical) syllogism always consists of two categorical premisses and a categorical conclusion to be inferred from the premisses. The two premisses share a class with the conclusion and share a common “middle” class that is not in the conclusion. The conclusion asserts some relationship between the classes it shares with the premisses.

Aristotle introduced the concept of syllogism as a way to reduce deductive reasoning to the study of *argument form*. The form of a syllogism refers to the type of premisses and conclusion that appear. If a particular argument form is valid, then all arguments having this form will be valid --- regardless of the specifics of the statements involved. (Remember the difference between “valid” and “sound” arguments.) In other words, an argument form is valid if we can substitute *any* statements into the form “template” and still produce a valid argument.

There are 256 forms of categorical syllogism; only twenty-four of these forms are templates for valid arguments. The propositional calculus has extended the notion of “argument form” well beyond the ancient syllogism concept; and, fortunately, provided us with a relatively simple means to determine when an argument form is the template for a valid argument.

### **Argument Form**

An *argument form* is a conditional statement having the structure

$$(p_1 \wedge \dots \wedge p_n) \rightarrow c$$

where  $p_1, \dots, p_n$  and  $c$  are variables allowed to denote logical statements. The variables  $p_1, \dots, p_n$  are called the *premiss* variables of the argument form, and the variable  $c$  is called the *conclusion* variable of the argument form. An argument form is valid provided it is a tautology.

It is important to note that we *do not* assume all (or even some) of the premiss variables are *simple*. If compound premiss variables exist in an argument form, then great care must be taken to maintain consistency when assigning logical statements to the variables.

**Problem 1.** You proved in the previous exercise set that the argument form  $([p \rightarrow q] \wedge p) \rightarrow q$  is valid. (This form is known as “Affirming the Antecedent” or *modus ponens*<sup>2</sup>. Does the following argument fit the “template” of this argument form? Explain.

- *If truckers love country music, then truckers enjoy honky-tonks.*
- *Truckers enjoy honky-tonks.*
- *Therefore, truckers love country music.*

<sup>1</sup> The word “syllogism” comes from the Greek verb “*syllogizesthai*” which means “to infer” (*syl-logizesthai* --- literally “to reckon with”).

<sup>2</sup> The phrase “*modus ponens*” means “mode that affirms by affirming.” It is believed Theophrastus first introduced this mode of argument.

**Problem 2.** The argument form  $([p \rightarrow q] \wedge [q \rightarrow r]) \rightarrow [p \rightarrow r]$  is known as as “Conditional Transitive Reasoning” or the “Hypothetical Syllogism.” You showed this argument form is valid in the previous set of exercises. Does the following valid argument fit this “template”? Explain carefully.

- *All women are things that are created equal.*
- *All things that are created equal are things that have certain inalienable rights.*
- *Therefore, all women are things that have certain inalienable rights.*

**Problem 3.** The argument form  $([p \rightarrow q] \wedge q) \rightarrow p$  is known as “Affirming the Consequent.”

**Part (a).** Construct an argument which fits this “template.”

**Part (b).** Is this argument form valid? Justify your answer.

**Problem 4.** The argument form  $([p \rightarrow q] \wedge \sim q) \rightarrow \sim p$  is known as “Denying the Consequent” or *modus tollens*<sup>3</sup>.

**Part (a).** Construct an argument which fits this “template.”

**Part (b).** Is this argument form valid? Justify your answer.

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<sup>3</sup> The phrase *modus tollens* means “mode that denies by denying.”

**Problem 5.** We know that both argument forms  $([p \rightarrow q] \wedge \sim q) \rightarrow \sim p$  and  $([r \rightarrow s] \wedge r) \rightarrow s$  are valid. Does this imply that the following argument form is also valid? Explain.

$$([p \rightarrow ([r \rightarrow s] \wedge r) \rightarrow s] \wedge \sim s) \rightarrow \sim p$$

**Exercises.**

Let  $p$  and  $q$  denote statement variables. In the previous investigation, you showed the following statement forms serve as a “template” for all categorical statements.

1. Universal Affirmative ---  $p \rightarrow q$
2. Universal Negative ---  $p \rightarrow \sim q$
3. Existential Affirmative ---  $p \wedge q$
4. Existential Negative ---  $p \wedge \sim q$

Construct an argument form that serves as a “template” for the following arguments. Use truth tables to decide which forms are valid.

**Problem 1.**

- *All leprechauns are green-eyed beings.*
- *No forest sprite is a non-(green-eyed being).*
- *Therefore, all leprechauns are forest sprites.*

**Problem 2.**

- *There exist politicians who are horses of a different color.*
- *There exist horses of a different color that are democrats.*
- *Therefore, there exist politicians who are democrats.*

**Problem 3.**

- *There exist reptiles that are snakes.*
- *All snakes are animals with scales.*
- *Therefore, there exist reptiles that are animals with scales.*

**Problem 4.** The *disjunctive syllogism* is another argument type that has been recognized since ancient times. It is represented by the argument form  $[(p \vee q) \wedge \sim q] \rightarrow p$ .

**Part (a).** Is this a valid argument form? Justify your answer.

**Part (b).** Construct an argument that fits the “template” of the disjunctive syllogism.

**Problem 5.** Is the following argument valid? Justify your answer.

- *Either someone left my cake out in the rain, or the sweet green frosting is not melting.*
- *No one left my cake out in the rain.*
- *Therefore, the sweet green frosting is not melting.*

**Problem 6.** Is the following argument valid? Justify your answer.

- *If someone left my cake out in the rain, then the sweet green frosting is not melting.*
- *No one left my cake out in the rain.*
- *Therefore, the sweet green frosting is not melting.*

**Problem 7.** A very observant student complained that there is an inconsistency between Problems 5 and 6. She said,

“Problem 5 is an example of the disjunctive syllogism and is valid. However, Problem 6 is also a version of the disjunctive syllogism since  $p \rightarrow q$  is equivalent to  $p \vee \sim q$ . How can one argument be valid and the other invalid if they are both examples of the same argument form?”

How would you respond to her question?