

The tables below show the group of triangle symmetries, along with a “mystery group” whose operation is defined by the second table.

Let $D_6 = (\{S_i, S_{1R}, S_{2R}, F S_i, F S_{1R}, F S_{2R}\}, *)$ represent the group of triangle symmetries (traditionally called the *dihedral group on six elements*).

- S_i --- Do nothing
- S_{1R} --- Rotate 120° clockwise
- S_{2R} --- Rotate 240° clockwise
- $F S_i$ --- Vertical flip
- $F S_{1R}$ --- Vertical flip followed by 120° clockwise rotation
- $F S_{2R}$ --- Vertical flip followed by 240° clockwise rotation

When filling in the table, apply the symmetry in Row *A* before the symmetry in Column *B*.

*	S_i	S_{1R}	S_{2R}	$F S_i$	$F S_{1R}$	$F S_{2R}$
S_i	S_i	S_{1R}	S_{2R}	$F S_i$	$F S_{1R}$	$F S_{2R}$
S_{1R}	S_{1R}	S_{2R}	S_i	$F S_{2R}$	$F S_i$	$F S_{1R}$
S_{2R}	S_{2R}	S_i	S_{1R}	$F S_{1R}$	$F S_{2R}$	$F S_i$
$F S_i$	$F S_i$	$F S_{1R}$	$F S_{2R}$	S_i	S_{1R}	S_{2R}
$F S_{1R}$	$F S_{1R}$	$F S_{2R}$	$F S_i$	S_{2R}	S_i	S_{1R}
$F S_{2R}$	$F S_{2R}$	$F S_i$	$F S_{1R}$	S_{1R}	S_{2R}	S_i

Let $G = (\{A, B, C, D, E, G\}, \odot)$ be the group whose operation is defined by the following table. (You may assume that G is a group.)

\odot	<i>A</i>	<i>B</i>	<i>C</i>	<i>D</i>	<i>E</i>	<i>G</i>
<i>A</i>	<i>B</i>	<i>A</i>	<i>D</i>	<i>C</i>	<i>G</i>	<i>E</i>
<i>B</i>	<i>A</i>	<i>B</i>	<i>C</i>	<i>D</i>	<i>E</i>	<i>G</i>
<i>C</i>	<i>G</i>	<i>C</i>	<i>B</i>	<i>E</i>	<i>D</i>	<i>A</i>
<i>D</i>	<i>E</i>	<i>D</i>	<i>A</i>	<i>G</i>	<i>C</i>	<i>B</i>
<i>E</i>	<i>D</i>	<i>E</i>	<i>G</i>	<i>A</i>	<i>B</i>	<i>C</i>
<i>G</i>	<i>C</i>	<i>G</i>	<i>E</i>	<i>B</i>	<i>A</i>	<i>D</i>

The group G is actually the “same” as the dihedral group D_6 .

TASK 1: On your own, think about how to demonstrate that these two groups are the same.

TASK 2: Share your thoughts with your group, come to a consensus, and demonstrate that these two groups are the same. What does “same” mean?