

When we have created a labeling scheme that proves two groups are the “same”, we call the labeling scheme an *isomorphism* and say that the two groups are *isomorphic*. (The terminology comes from the Greek phrase for “identical structure.”)

Consider the following labeling scheme between  $D_6$  and the mystery group  $G$ .

$$\begin{array}{lll} A \leftrightarrow {}_F S_i & B \leftrightarrow S_i & C \leftrightarrow {}_F S_{2R} \\ D \leftrightarrow S_{1R} & E \leftrightarrow {}_F S_{1R} & G \leftrightarrow S_{2R} \end{array}$$

TASK 1: Working on your own, decide whether or not this labeling scheme is an isomorphism.

TASK 2: Discuss and justify your decision with your group members.

TASK 3: Working on your own, decide whether or not it is possible to complete the labeling scheme started below so that the end result is an isomorphism.

$$G \leftrightarrow {}_F S_i \quad C \leftrightarrow S_{1R}$$

TASK 4: Discuss your strategy with your group members.

A labeling scheme between two groups is really a special *function* between the universes of these groups. For example, the labeling scheme used in Task 1 could be written using function notation.

Let  $L$  represent the labeling scheme as a function *from* the set  $G$  *to* the set  $D_6$ . In function notation, we would have

$$\begin{array}{lll} L(A) = {}_F S_i & L(B) = S_i & L(C) = {}_F S_{2R} \\ L(D) = S_{1R} & L(E) = {}_F S_{1R} & L(G) = S_{2R} \end{array}$$

TASK 5: Suppose we let  $M$  represent the labeling scheme as a function *from* the set  $D_6$  *to* the set  $G$ . Write the labeling scheme using function notation.

TASK 6: How are the functions  $L$  and  $M$  related to each other?

TASK 7: Suppose that  $\mathbf{G}=(G,*)$  and  $\mathbf{H}=(H,\#)$  are groups, and suppose  $f$  is a function *from* the set  $G$  *to* the set  $H$ . What conditions must  $f$  satisfy to guarantee that  $f$  is an isomorphism?