Let *EVENS* denote the set of all even integers, and let *ODDS* denote the set of all odd integers. Consider the combining rule defined on the set *P* = {*EVENS, ODDS*} according to the following table.

$\oplus$	***EVENS***	***ODDS***
EVENS	*EVENS*	*ODDS*
ODDS	*ODDS*	*EVENS*

TASK 1: What does it mean to write *EVENS* $\oplus$ *ODDS* = *ODDS*? Does (*P*, $\oplus$) form a group?

Suppose we break up the elements of the dihedral group $\boldsymbol{D}_8$ into the two sets

$$E = \{S_i, S_{1R}, S_{2R}, S_{3R}\} \qquad O = \{{}_FS_i, {}_FS_{1R}, {}_FS_{2R}, {}_FS_{3R}\}$$

TASK 2: If we arrange the operation table for $\boldsymbol{D}_8$ accordingly, what do you notice?

OFFICIAL SYMBOL	S_i	S_{1R}	S_{2R}	S_{3R}	${}_FS_i$	${}_FS_{1R}$	${}_FS_{2R}$	${}_FS_{3R}$
S_i	S_i	S_{1R}	S_{2R}	S_{3R}	${}_FS_i$	${}_FS_{1R}$	${}_FS_{2R}$	${}_FS_{3R}$
S_{1R}	S_{1R}	S_{2R}	S_{3R}	S_i	${}_FS_{3R}$	${}_FS_i$	${}_FS_{1R}$	${}_FS_{2R}$
S_{2R}	S_{2R}	S_{3R}	S_i	S_{1R}	${}_FS_{2R}$	${}_FS_{3R}$	${}_FS_i$	${}_FS_{1R}$
S_{3R}	S_{3R}	S_i	S_{1R}	S_{2R}	${}_FS_{1R}$	${}_FS_{2R}$	${}_FS_{3R}$	${}_FS_i$
${}_FS_i$	${}_FS_i$	${}_FS_{1R}$	${}_FS_{2R}$	${}_FS_{3R}$	S_i	S_{1R}	S_{2R}	S_{3R}
${}_FS_{1R}$	${}_FS_{1R}$	${}_FS_{2R}$	${}_FS_{3R}$	${}_FS_i$	S_{3R}	S_i	S_{1R}	S_{2R}
${}_FS_{2R}$	${}_FS_{2R}$	${}_FS_{3R}$	${}_FS_i$	${}_FS_{1R}$	S_{2R}	S_{3R}	S_i	S_{1R}
${}_FS_{3R}$	${}_FS_{3R}$	${}_FS_i$	${}_FS_{1R}$	${}_FS_{2R}$	S_{1R}	S_{2R}	S_{3R}	S_i

TASK 3: Suppose we break up the dihedral group into the following two sets.

$$A = \{S_i, {}_FS_i\ , {}_FS_{2R}\ , {}_FS_{3R}\} \qquad B = \{S_{1R}, S_{2R}, S_{3R}, {}_FS_{1R}\}$$

Do these subsets serve as "Evens" and "Odds" sets for $\boldsymbol{D}_8$? Justify your answer.

TASK 4: There are two other ways to break up $\boldsymbol{D}_8$ into "Evens" and "Odds" sets. Find at least one of these ways and arrange the operation table according to your sets.

Let $\boldsymbol{G} = (G,*)$ be a group, and suppose that A and B are nonempty subsets of G. We can define a combining rule for these sets in the following way:

$$A \circledast B = \{x * y : x \in A \text{ and } y \in B\}$$

TASK 6: Consider the sets

$$E = \{S_i, S_{1R}, S_{2R}, S_{3R}\} \qquad O = \{{}_FS_i, {}_FS_{1R}, {}_FS_{2R}, {}_FS_{3R}\}$$

from Task 2. Construct the sets $E \circledast O$ and $O \circledast E$.

TASK 7: Consider the sets

$$A = \{S_i, {}_FS_i\ , {}_FS_{2R}\ , {}_FS_{3R}\} \qquad B = \{S_{1R}, S_{2R}, S_{3R}, {}_FS_{1R}\}$$

from Task 3. Construct the sets $A \circledast B$ and $B \circledast A$.

TASK 8: Does the system $(\{E, O\}, \circledast)$ form a group? What about the system $(\{A, B\}, \circledast)$?