TASK 1: Take a look at one of the "Evens and Odds" groups you constructed from $\boldsymbol{D}_8$.

Part (a): Does every element of $\boldsymbol{D}_8$ appear in at least one of the "Evens" or "Odds" subsets?

Part (b): Does any element of appear in both the "Evens" and the "Odds"?

Part (c): Do you notice anything special about the "Evens" set?

A *partition* of a set *X* is a family of subsets from *X* with the property that every element of *X* is contained in *exactly one* member of the family.

Let ***G*** = (*G*,*) be a group. When you break up *G* into a partition that forms a group under the rule for combining subsets, you have formed a *quotient* group from ***G***.

TASK 2: Suppose you have created a quotient group for the group ***G***, and suppose that the subset *A* serves as the identity element for your quotient group.

Part (a): When you combine the set *A* with itself, what must be the result?

Part (b): Does this tell you anything about *A* and the operation * on *G*? Justify your thinking.

TASK 3: Again, suppose that the subset *A* serves as the identity element for your quotient group. Let *e* be the identity element for *G* and suppose that *B* is a member of the quotient group different from *A*.

Part (a): When you combine A and B, what must be the result?

Part (b): Explain why it is not possible for *e* to be a member of *B*.

TASK 4: Again, suppose that the subset *A* serves as the identity element for your quotient group and suppose that *B* is a member of the quotient group different from *A*. Now, suppose that *g* is an element of *A*. Use Task 2 (B) to explain why it is not possible for g^{-1} to be an element of *B*.

TASKS 3 and 4 tell us that the identity element of a quotient group from a group ***G*** must be a *subgroup* of ***G***.

To form a quotient group from ***G***, we need the identity element of the quotient group to be a subgroup of ***G***. So, if we start with a subgroup of ***G***, how can we determine what the other elements of the quotient group will be?

TASK 5: Look again at the "Evens" and the "Odds" from the integers under addition. The "Evens" set serves as the identity element of our quotient group. Suppose we pick any odd integer and add it to every member of the "Evens" set. By doing this, would we get every member of the "Odds" set? Justify your answer.