

TASK 1: How many possible combinations of two triangle symmetries are there?  
JUSTIFY!

Our official notation suggests a natural way to read combinations of symmetries:

Whenever we form the combination of two symmetries  $A$  and  $B$ , we will read the combination from left-to-right. It is customary to write  $A \cdot B$  to indicate that we perform symmetry  $A$  BEFORE we perform symmetry  $B$ .

TASK 3: Did you determine which of the six symmetries is equivalent to your combinations by manipulating the triangle around or by using shortcuts?

TASK 4: When working on your combinations, did you ever run into a need for the “do nothing” transformation? If so, how did the need come up?

TASK 5: If you or someone in your group used shortcuts, what are they? Can your group justify why the shortcuts might work? Write your shortcuts using the official class notation for combinations.

TASK 6: Fill in the table below, this time using only shortcuts. Do not move the triangle around. Keep track of the shortcuts you use, especially if you develop any that are different from your list above.

When filling in the table, apply the symmetry in Row *A* before the symmetry in Column *B*.

Official Symbol	$S_i$	$S_{1R}$	$S_{2R}$	$FS_i$	$FS_{1R}$	$FS_{2R}$
$S_i$						
$S_{1R}$						
$S_{2R}$						
$FS_i$						
$FS_{1R}$						
$FS_{2R}$						

TASK 7: Write down all of the shortcuts your group used to fill in the table. Whenever possible, write your shortcuts as equations using combinations of our symbol for a flip about the vertical axis and our symbol for clockwise rotation through  $120^\circ$ .

TASK 8: Here is a list of shortcuts. If these shortcuts are not already on your list, verify that they are valid.

### LIST OF SHORTCUTS

1. Combining symmetries is associative; that is, for any symmetries  $A$ ,  $B$ , and  $C$ , we always have  $A(BC) = (AB)C$ .
2. Combining the do-nothing transformation with any symmetry  $A$  gives us the symmetry  $A$ .
3. We have  $S_{1R} \cdot S_{1R} \cdot S_{1R} = S_i$ , and we have  ${}_F S_i \cdot {}_F S_i = S_i$ .
4. We have  $S_{1R} \cdot {}_F S_i = {}_F S_i \cdot S_{1R} \cdot S_{1R}$ .
5. We have  ${}_F S_i \cdot S_{1R} = S_{1R} \cdot S_{1R} \cdot {}_F S_i$ .
6. We have  $S_{1R} \cdot {}_F S_i \cdot S_{1R} = {}_F S_i$ .
7. We have  ${}_F S_i \cdot S_{1R} \cdot {}_F S_i \cdot S_{1R} = S_i$ .

TASK 9: Some of the shortcuts on this list are unnecessary, because they can be derived from others on the list.

GROUP 1: Show that we could keep Shortcuts 1 – 6 and eliminate Shortcut 7.

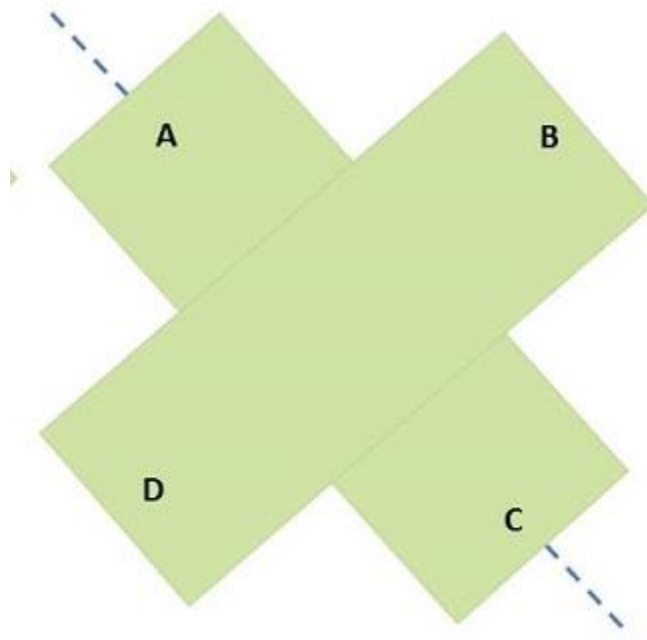
GROUP 2: Show that we could keep Shortcuts 1 – 4 and 6 – 7 and eliminate Shortcut 5.

GROUP 3: Show that we could keep Shortcuts 1 – 5 and 7 and eliminate Shortcut 6.

GROUP 4: Show that we could keep Shortcuts 1 - 2 and 4 – 7 and eliminate Shortcut 3.

*Be sure to indicate clearly which Shortcuts you apply when deriving one shortcut from others.*

TASK 10: Let's return to the symmetries of the plus-sign you worked with in Activity 1.



There are eight distinct symmetries of this figure. Using the official symbol scheme we adopted for the equilateral triangle, these distinct symmetries can be named

$S_i$  --- Do nothing

$S_{1R}$  --- Rotate  $90^\circ$  clockwise

$S_{2R}$  --- Rotate  $180^\circ$  clockwise

$S_{3R}$  --- Rotate  $270^\circ$  clockwise

$FS_i$  --- Diagonal flip

$FS_{1R}$  --- Diagonal flip followed by  $90^\circ$  clockwise rotation

$FS_{2R}$  --- Diagonal flip followed by  $180^\circ$  clockwise rotation

$FS_{3R}$  --- Diagonal flip followed by  $270^\circ$  clockwise rotation

Using the notation conventions we agreed to for the symmetries of the equilateral triangle, fill in the table on the next page to show how every combination of symmetries is equivalent to one of those listed.

OFFICIAL SYMBOL	$S_i$	$S_{1R}$	$S_{2R}$	$S_{3R}$	$FS_i$	$FS_{1R}$	$FS_{2R}$	$FS_{3R}$
$S_i$								
$S_{1R}$								
$S_{2R}$								
$S_{3R}$								
$FS_i$								
$FS_{1R}$								
$FS_{2R}$								
$FS_{3R}$								

TASK 11: As you filled in the table, did you notice any shortcuts similar to the ones you found for combining symmetries of the equilateral triangle? What are some of them?