TASK 1: In your group, check your tables that summarize all the combinations of symmetries for the plus-sign. Correct your table if necessary.

TASK 2: In your group, discuss your derivations from Task 9 of Activity 2 and come to a group consensus.

GROUP CONSENSUS:

Shortcut that can be eliminated while keeping the others:

Proof that this shortcut can be derived from others on the list:

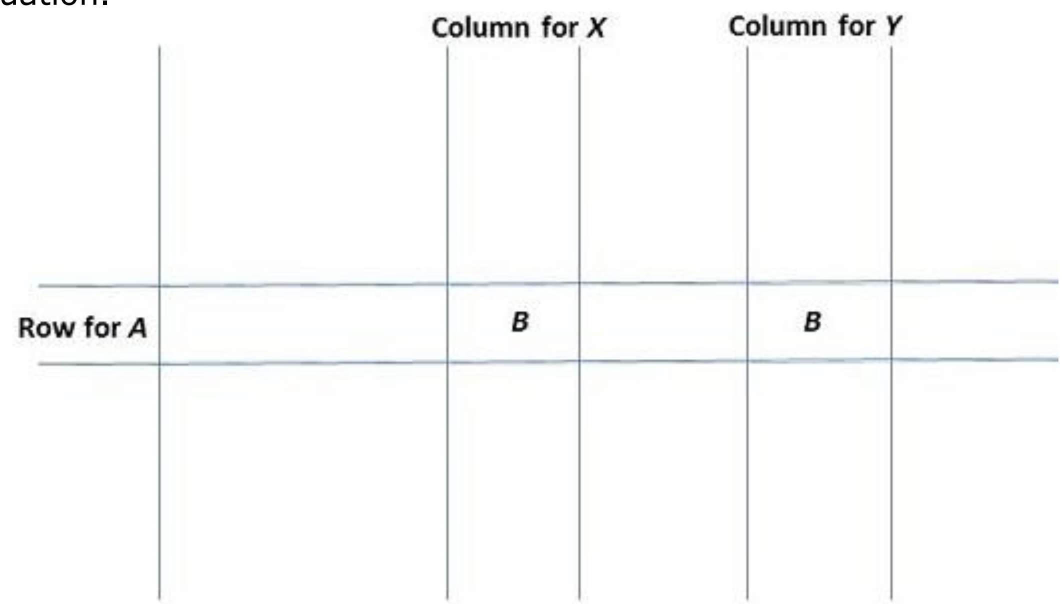
Now, take a look at the tables you made that summarize the combinations of symmetries for the equilateral triangle and the plus-sign. Notice that each symmetry appears exactly once in each non-shaded row and exactly once in each non-shaded column. We will refer to this as the "***Sudoku property***."

Our next task will be to see if the shortcuts you explored in Tasks 8 and 9 are sufficient to *prove* the Sudoku property for the triangle table; and hopefully do so in such a way that we can conclude this property holds for *any* collection of symmetries.

We have to be particularly careful about this, because there are some objects that possess an *infinite* number of symmetries. (Consider the symmetries of a circle, for example.)

First, we are going to prove that every symmetry can appear AT MOST ONCE in each row of the table --- without checking every row of the table directly.

TASK 3: Suppose that *A* and *B* are any two of the six symmetries of the triangle. Suppose that *B* appears twice in the row for symmetry *A*. Using our notation for combinations of symmetries, write two equations that represent this situation.



Portion of the Symmetry Table

TASK 4: What can you conclude from these equations? Be careful! Did you use a property of the symmetries that is not represented on our list from Task 8 of Activity 3?

TASK 5: Based on your work in Tasks 3 and 4, how could you phrase the property "Every symmetry appears at most once in the row for Symmetry *A*" so that it still makes sense for an *infinite* set of symmetries?

TASK 6: In order to prove that any symmetry *B* appears at most one time in any row of the table, we need another property that is not in the list from Task 8 of Activity 3. (It might be on your list, however.) State this new property in a complete sentence and explain how it is used along with the equations from Task 3 to prove the result.

In order to know that every symmetry appears *exactly once* in each row of the symmetry table, we still need to prove that each one appears AT LEAST ONCE in each row.

TASK 7: As long as we are dealing with a *finite* collection of symmetries, knowing that each symmetry appears *at most once* in each row is enough to allow us to conclude that that each symmetry must also appear *at least once* in each row. Why?

TASK 8: Think about our answer for Task 5 above. How could you rephrase the property "Every symmetry appears at least once in the row for Symmetry *A*" so that it still makes sense for an *infinite* set of symmetries?

TASK 9: Devise a proof that your rephrased property holds.