Let $\boldsymbol{Z}_n$ represent the system consisting of the set {0,1,2,…,n-1} along with the following rule for combining pairs of members from this set:

$x \boxplus_n y$ is the remainder obtained when you divide $x + y$ by *n*

TASK 1: Compute all distinct combinations of two members from the system $\boldsymbol{Z}_3$ by filling in the table below.

$\boxplus_3$	0	1	2
0			
1			
2			

TASK 2: Compute all distinct combinations of two members from the system $\boldsymbol{Z}_4$ by filling in the table below.

$\boxplus_4$	0	1	2	3
0				
1				
2				
3				

TASK 3: Using your tables, compare the systems $\boldsymbol{Z}_3$ and $\boldsymbol{Z}_4$ to the systems of symmetries for the equilateral triangle and the plus-sign under their rule for combining pairs of elements.

PART A: What are some properties that are NOT shared by all four systems?

PART B: What are some properties that ARE shared by all four systems?

TASK 4: Consider the system $\boldsymbol{Z}$ of integers under the combining rule of ordinary addition. Does this system share any properties in common with the other four systems?

TASK 5: Consider the system $\boldsymbol{M}_2$of all nonsingular (invertible) matrices with real number entries under the combining rule of matrix multiplication. Does this system share any properties in common with the other five systems?

A *group* is a system that satisfies all of the properties shared by each of the six systems mentioned above.

TASK 6: Identify another system that is a group different from the systems mentioned above. Explain how your system satisfies the shared properties.

TASK 7: Carefully write down a definition that explains what a group is. Your definition should be a complete sentence, should include what is meant by the term "*system*," and should a list of the shared properties.

TASK 8: Consider the set {-1, 0, 1}.

PART A: Does this set form a group under the combining rule of integer addition?

PART B: Does this set form a group under the combining rule of integer multiplication?

TASK 9: Let $S = \{5, 15, 25, 35\}$. Consider the combining rule

$x \boxtimes_{40} y$ is the remainder obtained when you divide xy by 40

PART A: Fill in the table below.

$\boxtimes_{40}$	5	15	25	35
5				
15				
25				
35				

PART B: Does the set *S* form a group under this combining rule? Justify your answer.