

TASK 1: Is it possible for a group to possess more than one identity element? Justify your answer with an example if it is possible, or with a proof if it is not possible.

TASK 2: Devise a *cancellation law* that holds in all groups. Prove your law using the group axioms.

TASK 3: Let  $\mathbf{Q}^*$  represent the set of all rational numbers *except for the number -1*. Consider the combining rule

$$a \bowtie b = a + ab + b$$

PART A: Show that this combining rule is an operation on the set  $\mathbf{Q}^*$ . (It helps to notice that  $a \bowtie b = a(1 + b) + b$ . What happens if you assume  $a \bowtie b = -1$ ?)

PART B: Show that this combining rule is associative.

PART C: Show that this combining rule is commutative.

PART D: Show that  $\mathbf{Q}^*$  has an identity element under this combining rule.

PART E: Show that every member of  $\mathbf{Q}^*$  has an inverse under this combining rule.