FIRST GRADED HOMEWORK ASSIGNMENT

Consider the six distinct symmetries of the equilateral triangle. Here are three relations involving the symmetries S_{1R} and ${}_FS_i$.

1. ${}_FS_i \cdot S_{1R} \cdot {}_FS_i \cdot S_{1R} = S_i$
2. $S_{1R} \cdot {}_FS_i \cdot S_{1R} = {}_FS_i$
3. ${}_FS_i \cdot S_{1R} \cdot {}_FS_i = S_{1R} \cdot S_{1R}$

Show that each one of these relations implies the other two. In other words, assume only Relation 1 is true, and use it to derive the other two. Then, assume only Relation 2 is true, and use it to derive the other two, and so on.

You may use associativity and the following rules, but nothing else.

> We have $S_{1R} \cdot S_{1R} \cdot S_{1R} = S_i$, we have ${}_FS_i \cdot {}_FS_i = S_i$, and we have $A \cdot S_i = A = S_i \cdot A$ for all symmetries *A*.

Do your work on a separate sheet of paper. You will have six derivations when you are done. For each set of three derivations, clearly indicate which relation you are assuming is true and which ones you are deriving. Be sure to clearly indicate where you use associativity, and when you make use of your assumed relation.

EXAMPLE: Showing that

$${}_FS_i \cdot S_{1R} \cdot {}_FS_i = S_{1R} \cdot S_{1R} \text{ implies } S_{1R} \cdot {}_FS_i \cdot S_{1R} \cdot {}_FS_i \cdot S_{1R} = S_{1R}$$

PROOF: We know

$S_{1R} \cdot {}_FS_i \cdot S_{1R} \cdot {}_FS_i \cdot S_{1R}$	$= S_{1R} \cdot ({}_FS_i \cdot S_{1R} \cdot {}_FS_i) \cdot S_{1R}$	Associativity
	$= S_{1R} \cdot (S_{1R} \cdot S_{1R}) \cdot S_{1R}$	Assumption
	$= S_{1R} \cdot (S_{1R} \cdot S_{1R} \cdot S_{1R})$	Associativity
	$= S_{1R} \cdot S_i$	Assumption
	$= S_{1R}$	Assumption