ELEVENTH GRADED HOMEWORK ASSIGNMENT

Suppose that ***G***=(G,*) and ***H*** = (H, #) are groups, and suppose *f* is a function *from* the set *G to* the set *H*. The function *f* is an *isomorphism* provided the following conditions are met:

- The function *f* is a bijection.
- For all $a, b \in G$, it is the case that $f(a * b) = f(a) \# f(b)$.

Two groups are *isomorphic* provided there is an isomorphism between them.

Problem 1. Let $\boldsymbol{Z} = (\mathbb{Z}, +)$ be the group of integers under addition. For a fixed positive integer *n*, let $n\boldsymbol{Z} = (n\mathbb{Z}, +)$ be the group of integer multiples of *n* under addition. Prove that these groups are isomorphic.

Problem 2. Let $\boldsymbol{R} = (\mathbb{R}, +)$ be the group of real numbers under addition and also let $\boldsymbol{R}^+ = (\mathbb{R}^+, \cdot)$ be the group of positive real numbers under multiplication. Show that the function $f: \boldsymbol{R} \rightarrow \boldsymbol{R}^+$ defined by $f(x) = e^x$ is an isomorphism, where *e* represents euler's constant.