TWELFTH GRADED HOMEWORK ASSIGNMENT

The group $\wp_3 = (P_3\,,\ \circ)$ contains exactly six elements, namely the functions

$$e : \begin{pmatrix} 1 & 2 & 3 \\ 1 & 2 & 3 \end{pmatrix} \qquad f : \begin{pmatrix} 1 & 2 & 3 \\ 1 & 3 & 2 \end{pmatrix} \qquad g : \begin{pmatrix} 1 & 2 & 3 \\ 2 & 1 & 3 \end{pmatrix}$$

$$h : \begin{pmatrix} 1 & 2 & 3 \\ 3 & 2 & 1 \end{pmatrix} \qquad j : \begin{pmatrix} 1 & 2 & 3 \\ 2 & 3 & 1 \end{pmatrix} \qquad k : \begin{pmatrix} 1 & 2 & 3 \\ 3 & 1 & 2 \end{pmatrix}$$

Problem 1: Construct an isomorphism from the dihedral group $\boldsymbol{D}_6$ to the group $\wp_3$. You must prove that your function is an isomorphism. (It helps to number the vertices of the triangle.)

Under matrix multiplication, the set below forms a cyclic group:

$$G = \left\{ \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}, \begin{bmatrix} 0 & -1 \\ 1 & 1 \end{bmatrix}, \begin{bmatrix} -1 & -1 \\ 0 & 1 \end{bmatrix}, \begin{bmatrix} -1 & 0 \\ 0 & -1 \end{bmatrix}, \begin{bmatrix} 0 & 1 \\ -1 & -1 \end{bmatrix}, \begin{bmatrix} 1 & 1 \\ -1 & 0 \end{bmatrix} \right\}$$

(You do not need to prove this.) The matrix

$$A = \begin{bmatrix} 0 & -1 \\ 1 & 1 \end{bmatrix}$$

serves as a generator for the group ***G***=(G,*), where * represents matrix multiplication.

Problem 2. Construct and isomorphism from the group ***G*** to the group $\boldsymbol{Z}_6 = (\mathbb{Z}_6,\ \boxplus_6)$ under addition modulo 6. You will need to prove that your function is an isomorphism. (In creating the function, it helps to write each member of *G* as a power of *A*.)