THIRTEENTH GRADED HOMEWORK ASSIGNMENT

Problem 1. Suppose that ***G*** = (G,*) and ***H*** = (H, #) are isomorphic groups, and suppose that $f: G \longrightarrow H$ is an isomorphism. Let *n* be a positive integer. Use Task 3 (B) and (C) of Activity 3 to prove $f(a^{-n}) = [f(a)]^{-n}$. (Therefore, *f* preserves *all* powers of *a*.)

Problem 2. Suppose that ***G*** = (G,*) and ***H*** = (H, #) are isomorphic groups, and suppose that $f: G \longrightarrow H$ is an isomorphism from ***G*** to ***H***. Suppose that $a \in G$, has order *n*.

Part (a) Use Task 3 (A) and (C) of Activity 3 to help show that the order of *f*(*a*) is at most *n*.

Part (b) Let *m* be the order of *f*(*a*) and let $g: H \longrightarrow G$ be the inverse function for *f*. Use the fact that *g* is also an isomorphism, along with Task 3 (A) and (C), to show that the order of *a* is at most *m*. (Therefore, we must have *n* = *m*.)

Problem 2 tells us that an isomorphism between groups must preserve the order of elements that have finite order. (It must preserve infinite order as well, but a different proof is needed for this.)

Problem 3. Suppose that ***G*** = (G,*) is an infinite cyclic group, and suppose that *a* is a generator for ***G***. We know

$$G = \{a^n : n \in \mathbb{Z}\}$$

Use this fact to construct an isomorphism from the group $\boldsymbol{Z} = (\mathbb{Z}, +)$ of integers under addition to the group ***G***. You will need the results from Homework Assignment 8 to prove that your function is an isomorphism.

Problem 4: Explain why the dihedral group $\boldsymbol{D}_6$ is NOT isomorphic to the group $\boldsymbol{Z}_6$.

Problem 5: Explain why the Quaternion Group is NOT isomorphic to the dihedral group $\boldsymbol{D}_8$ of symmetries for the plus-sign.