FOURTEENTH GRADED HOMEWORK ASSIGNMENT

Suppose that $\boldsymbol{G} = (G,*)$ and $\boldsymbol{H} = (H,\#)$ are groups. The *product group* $\boldsymbol{G} \times \boldsymbol{H}$ is defined to be the Cartesian product $G \times H$ along with the combining rule

$$(a, b) \otimes (c, d) = (a * c, b \# d)$$

In other words, the product of two ordered pairs is created by forming the product of the coordinates of each pair in their respective groups. This definition can be extended to any finite collection of groups.

As an example, consider the group $\boldsymbol{Z}_3 = (\mathbb{Z}_3, \boxplus_3)$ and the group $\boldsymbol{Z}_2 = (\mathbb{Z}_2, \boxplus_2)$. The product group $\boldsymbol{Z}_3 \times \boldsymbol{Z}_2$ has the following operation table.

$\otimes$	(0,0)	(1,0)	(2,0)	(0,1)	(1,1)	(2,1)
(0,0)	(0,0)	(1,0)	(2,0)	(0,1)	(1,1)	(2,1)
(1,0)	(1,0)	(2,0)	(0,0)	(1,1)	(2,1)	(0,1)
(2,0)	(2,0)	(0,0)	(1,0)	(2,1)	(0,1)	(1,1)
(0,1)	(0,1)	(1,1)	(2,1)	(0,0)	(1,0)	(2,0)
(1,1)	(1,1)	(2,1)	(0,1)	(1,0)	(2,0)	(0,0)
(2,1)	(2,1)	(0,1)	(1,1)	(2,0)	(0,0)	(1,0)

For example, $(1,1) \otimes (2,0) = (1 \boxplus_3 2\,, 1 \boxplus_2 0) = (0,1)$.

Problem 1: In the product group $\boldsymbol{Z}_3 \times \boldsymbol{Z}_2$, what is the order of the following elements?

(2,0) (0,1) (2,1)

Problem 2: Is the group $\boldsymbol{Z}_3 \times \boldsymbol{Z}_2$ cyclic? Explain your answer.

Problem 3: Construct the operation table for the product group $\boldsymbol{Z}_2 \times \boldsymbol{Z}_4$.

Problem 4: Construct the operation table for the product group $\boldsymbol{Z}_2 \times \boldsymbol{Z}_2 \times \boldsymbol{Z}_2$.

Problem 5: Is the group $\boldsymbol{Z}_2 \times \boldsymbol{Z}_4$ isomorphic to the group $\boldsymbol{Z}_2 \times \boldsymbol{Z}_2 \times \boldsymbol{Z}_2$? Justify your answer.