FIFTEENTH GRADED HOMEWORK ASSIGNMENT

Problem 1. Let $\boldsymbol{M}_2$ represent the group of all invertible 2x2 matrices under the operation of matrix multiplication, and let $\wp_4$ represent the group of all permutations on a four-element set under the operation of function composition.

Part (a): What is the inverse of $\left(\begin{bmatrix} 1 & 3 \\ 4 & 10 \end{bmatrix},\ (123)(34)\right)$ in the product group $\boldsymbol{M}_2 \times \wp_4$?

Part (b): What is the order of $\left(\begin{bmatrix} 1 & -1 \\ 0 & 1 \end{bmatrix},\ (123)\right)$ in the product group $\boldsymbol{M}_2 \times \wp_4$?

Problem 2: Suppose that $\boldsymbol{G} = (G,*)$ and $\boldsymbol{H} = (H,\#)$ are groups.

Part (a): If e is the identity element for $\boldsymbol{G}$ and i is the identity element for $\boldsymbol{H}$, prove that $(e\ ,\ i\ )$ is the identity element for the product group $\boldsymbol{G} \times \boldsymbol{H}$.

Part (b): Suppose that $a \in G$ and $b \in H$. Prove that the inverse for (a, b) in the product group $\boldsymbol{G} \times \boldsymbol{H}$ is the element (a^{-1}, b^{-1}), where a^{-1} is the inverse of a in $\boldsymbol{G}$ and b^{-1} is the inverse of b in $\boldsymbol{H}$.

Problem 3: The element (2,3) is a generator for the product group $\boldsymbol{Z}_3 \times \boldsymbol{Z}_4$.

Part (a): Write all twelve elements of this group as a power of (2,3).

Part (b): Use Part (a) to construct an isomorphism from $\boldsymbol{Z}_3 \times \boldsymbol{Z}_4$ to the group $\boldsymbol{Z}_{12}$. You must prove your mapping is an isomorphism. Try not to use the method of exhaustion.

Problem 4: The product group $\boldsymbol{Z}_2 \times \boldsymbol{Z}_6$ is NOT isomorphic to the group $\boldsymbol{Z}_{12}$. Find at least one reason why.