

## SECOND GRADED HOMEWORK ASSIGNMENT

Consider the eight distinct symmetries of the plus-sign. Recall that in this setting, we let  ${}_FS_i$  represent a flip of the figure about the main diagonal, and we let  $S_{1R}$  represent one  $90^\circ$  rotation clockwise. Combining symmetries of the plus-sign is associative, and we know the following two power relations are true

$$(S_{1R})^4 = S_{1R} \cdot S_{1R} \cdot S_{1R} \cdot S_{1R} = S_i \text{ and } ({}_FS_i)^2 = {}_FS_i \cdot {}_FS_i = S_i$$

Here are three additional relations involving these two symmetries.

1.  ${}_FS_i \cdot S_{1R} \cdot S_{1R} \cdot S_{1R} = S_{1R} \cdot {}_FS_i$
2.  $S_{1R} \cdot {}_FS_i \cdot S_{1R} = {}_FS_i$
3.  ${}_FS_i \cdot S_{1R} \cdot {}_FS_i \cdot S_{1R} = S_i$

Each of these relations can be verified directly by manipulating the plus-sign. However, using only associativity and the two power relations above, prove that these three relations are logically equivalent. This exercise involves three steps.

- First, assuming associativity, the two power relations, and Relation 1 are valid, show that you can derive Relations 2 and 3.
- Second, assuming associativity, the two power relations, and Relation 2 are valid, show you can derive Relations 1 and 3.
- Third, assuming associativity, the two power relations, and Relation 3 are valid, show you can derive Relations 1 and 2.