THIRD GRADED HOMEWORK ASSIGNMENT

You will probably need to use outside sources to help you answer Problem 4. If you use outside sources, you must cite them, and you must write the solution in your own words, using the notation given here.

Problem 1. Explain why it is not correct to say "the set {0,1,2,3,4,5,6,7} forms a group." What else needs to be said?

Problem 2. Consider the set Q^* of all nonzero rational numbers under the combining rule

$$a \uplus b = \frac{a}{b} + \frac{b}{a}$$

Part A: Give a specific example to show that this rule is not associative; in other words, use specific values of a, b, and c to show that it is possible to have

$$(a \uplus b) \uplus c \neq a \uplus (b \uplus c)$$

Part B: Prove that this rule is commutative; that is, prove $a \uplus b = b \uplus a$.

Problem 3. Construct the table that shows all possible distinct combinations of two members from the group $\mathbf{Z}_8$.

Problem 4. Consider the system $\mathbf{M}_2$ of all nonsingular (invertible) matrices with real number entries under the combining rule of matrix multiplication. (In linear algebra, you showed that this combining rule is associative, but is not commutative.) What is the inverse for the element
$$\begin{bmatrix} a & b \\ c & d \end{bmatrix}$$

Prove that your element does indeed serve as the inverse for this matrix.