SIXTH GRADED HOMEWORK ASSIGNMENT

DO NOT USE AN OUTSIDE SOURCE FOR THIS ASSIGNMENT.

Problem 1. Consider the group $\boldsymbol{M}_2$ of all nonsingular 2x2 matrices with real number entries under the operation of matrix multiplication. What is the order of each of the following matrices in this group?

$$A = \begin{bmatrix} -1 & -1 \\ 1 & 0 \end{bmatrix} \qquad B = \begin{bmatrix} 1 & 2 \\ 0 & 1 \end{bmatrix}$$

Problem 2. Suppose that $\boldsymbol{G} = (G,*)$ is a group with identity $\boldsymbol{e}$, and suppose that *a* is a member of *G* having order *n*. Explain why it is not possible to have $a^j = \boldsymbol{e}$ for any positive integer *j* < *n*.

The Division Algorithm tells us that for any fixed positive integer *n* and any integer *m*, there exist unique integers *q* and *r* such that

$$0 \leq r < n \quad \text{and} \quad m = nq + r$$

Problem 3. Suppose that $\boldsymbol{G} = (G,*)$ is a group with identity $\boldsymbol{e}$, and suppose that *a* is a member of *G* having order *n*. Use the Division Algorithm and the laws of exponents to prove the following conjectures.

PART A: If *n* < *m*, then there exists some $0 \leq r < n$ such that $a^m = a^r$.

PART B: We have $a^m = \boldsymbol{e}$ if and only if *m* is an integer multiple of *n*.