SEVENTH GRADED HOMEWORK ASSIGNMENT

A group $\boldsymbol{G} = (G,*)$ is *cyclic* provided every member of *G* is a (possibly negative) power of some fixed member of *G*. This fixed member is called a *generator* for the group $\boldsymbol{G}$.

For example, 3 is a generator for the group $\boldsymbol{Z}_8$. To see why, observe

$$3^1 = 3 \qquad 3^2 = 3 \boxplus_8 3 = 6 \qquad 3^3 = 3 \boxplus_8 3 \boxplus_8 3 = 1$$

$$3^4 = 3^3 \boxplus_8 3 = 4 \qquad 3^5 = 3^4 \boxplus_8 3 = 7 \qquad 3^6 = 3^5 \boxplus_8 3 = 2$$

$$3^7 = 3^6 \boxplus_8 3 = 5 \qquad 3^8 = 3^7 \boxplus_8 3 = 0$$

We see that every element of the group $\boldsymbol{Z}_8$ is a power of the element 3. Therefore, is $\boldsymbol{Z}_8$ cyclic.

Cyclic groups can have many different generators.

Problem 1. What are the generators for the group $\boldsymbol{Z}_8$? Justify your answer.

Let *K* = {*e, t, u, v, w, x, y, z*}. The set *K* forms a group under the combining rule defined by the table below. (You don't need to prove this.) The system $\boldsymbol{K} = (K,\circledast)$ is called the *Quaternion Group*.

$\circledast$	e	t	u	v	w	x	y	z
e	e	t	u	v	w	x	y	z
t	t	e	v	u	x	w	z	y
u	u	v	t	e	y	z	x	w
v	v	u	e	t	z	y	w	x
w	w	x	z	y	t	e	u	v
x	x	w	y	z	e	t	v	u
y	y	z	w	x	v	u	t	e
z	z	y	x	w	u	v	e	t

Problem 2. Use the table to determine the order of each member of the Quaternion Group.

Problem 3. Is the Quaternion Group cyclic? Justify your answer.

Problem 4. Is the group $\boldsymbol{Z} = (\mathbb{Z}, +)$ of integers under ordinary addition cyclic? Justify your answer.