EIGHTH GRADED HOMEWORK ASSIGNMENT

Problem 1. Let $\boldsymbol{G} = (G,*)$ be a group with identity element ***e***, and suppose that $a \in G$. Suppose that *j* and *k* are positive integers such that *j* < *k*.

PART A: If $a^j = a^k$, what can we say about the element a^{k-j} ? Justify your answer.

PART B: If $a^j = a^k$, use your answer to Part A to explain why the order of the element *a* must be strictly less than *k*.

Problem 2. Let $\boldsymbol{G} = (G,*)$ be a group with identity element ***e***, and suppose that $a \in G$ has order *n*. Use Problem 1 (B) to explain why it is not possible to have $a^j = a^k$ whenever $0 \leq j < k < n$.

Problem 3. Let $\boldsymbol{G} = (G,*)$ be a group with identity element ***e***, and suppose that $a \in G$. In this exercise, we address the following question.

Suppose *x* and *y* are positive integers. When does $a^x = a^y$ guarantee that *x* = *y*?

PART A: If *a* has infinite order, use Problem 1 (B) to prove that $a^x = a^y$ always guarantees that *x* = *y*.

PART B: Suppose *a* has finite order *n* and suppose that *x* and *y* are positive integers such that that $kn \leq x, y < (k+1)n$ for some integer *k*. (That is, *x* and *y* are both between consecutive integer multiples of *n*.)

Part (i) Use the Division Algorithm to show there exist integers *r, s* such that

- $0 \leq r, s < n$
- $x = kn + r$ and $y = kn + s$

Part (ii) Use Problem 3 (A) from Homework Assignment 6 along with Problem 2 from this assignment to show that $a^x = a^y$ implies *x* = *y*.